MACHINE DESIGN

Design - is a process plan to satisfy a human need.
It is a mechanism to convert an idea into reality
It is a solution to the problem.

Machnine - A m/c is a combination of resistant bodies with successfully constrained motion (relative motion) to other form of energy into mechanical energy.

OR

To transmit and modify the available energy to do some useful work.

Machine design involve the selection of sotable material and optimum dimentions to withstand the load without failure.
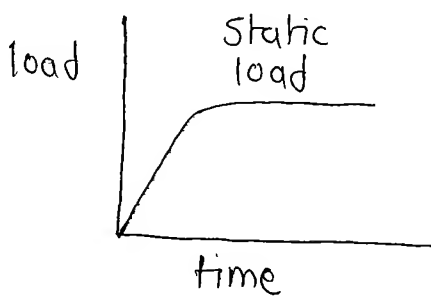
Loads - Externally applied force or moment acting on the member

loads

Static ← → Variable load

Cont.
w.r to
time
{ Magnitude
Direction
Point of application

load | Static load

time
(a)
Gradually applied load
Can also called as static load.

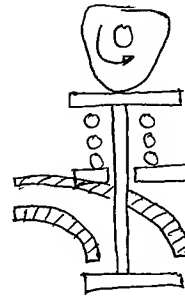load | Suddenly applied load.

time
(b)
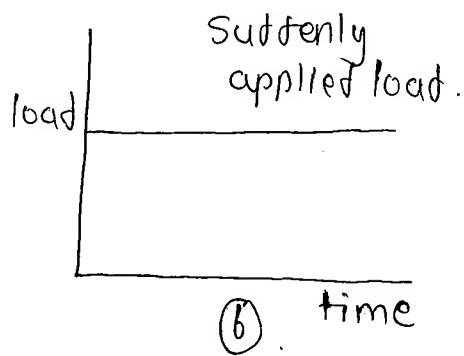
① Magnitute changes - magnitute is fluxituating but direction remain same
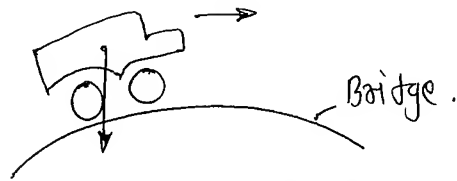
EX:-

the direction of force is cont· but magnitute is changing while vehicle crossing the bridge.
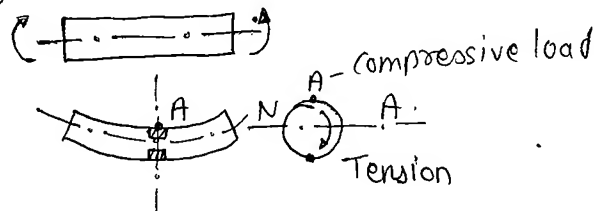
Bridge.

valve spring.

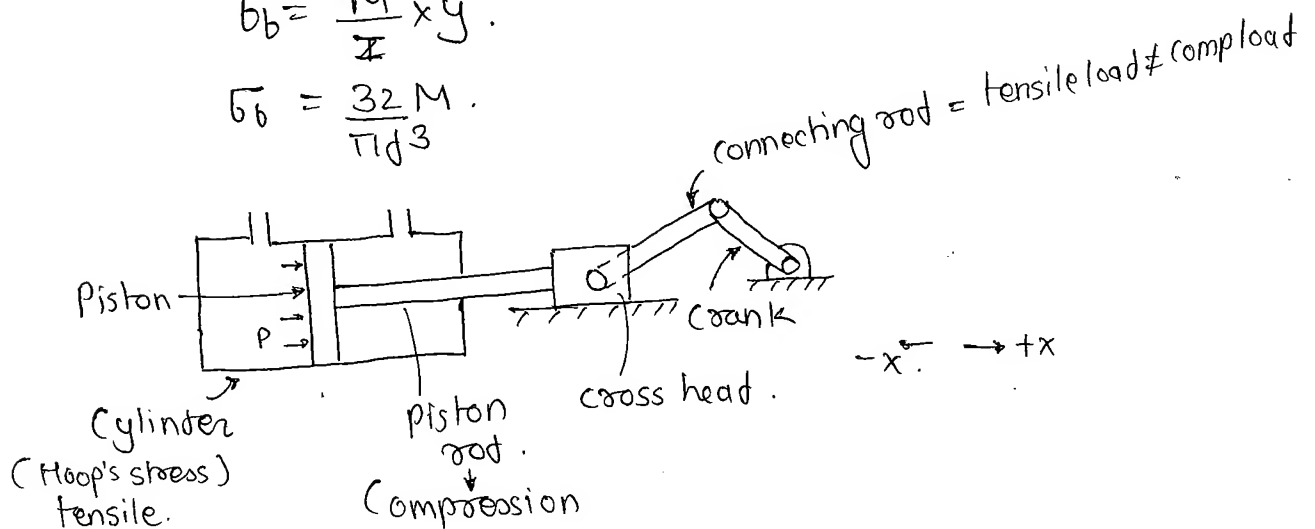② Direction Changes - Constant magnitute

Ex:- Rack pinion·, gear m/csm.
A rotating shaft carrying a const· bending movement·

A - compressive load

Tension

$$\overline{6}b = \frac{M}{I} \times y.$$

$$\overline{6}b = \frac{32 M}{\pi d^3}$$

connecting rod = tensile load ≠ comp load

Piston

P

Cylinder
(Hoop's stress)
tensile.

piston rod·
↓
Compression

Crank

cross head·

$-x \leftarrow \longrightarrow +x$

Pressure P

$$\vec{F} = \text{Pres} \times \text{Area}$$
$$= P \times \frac{\pi}{4} D^2.$$
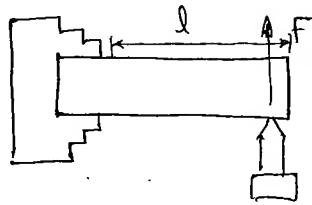$$\overleftarrow{F} = P \times \frac{\pi}{4} (D^2 - d^2)$$

$$d << D.$$

$$\vec{F} = \overleftarrow{F}$$

Piston rod = $\vec{F}$ - tensile load $\vec{F}$ - comp· load·

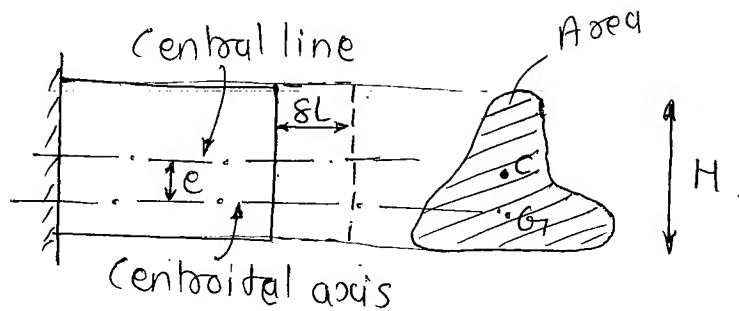hence piston rod come under this catagory·

③ Point of application.

$$M_{max} = F \times l$$
$$M_{min} = F \times o$$

Stress (σ)

$$\sigma = \frac{load}{area} = \frac{F}{A}$$

centroidal axis.

$$\sigma_{central\ line} = \frac{F}{A} + \frac{M}{I}y$$

Strain (ε)
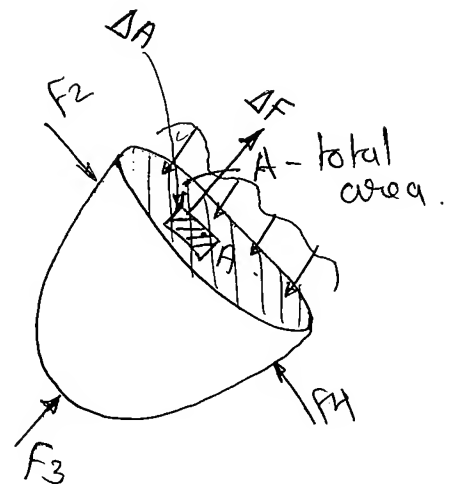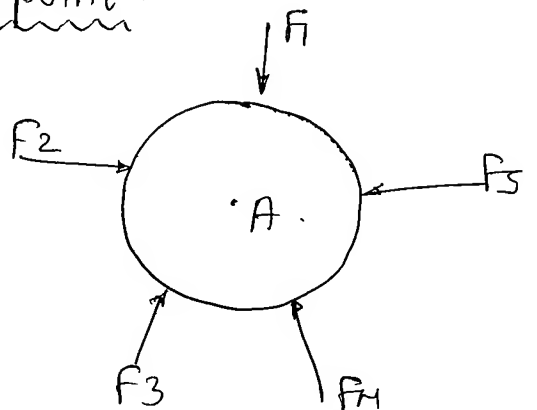
$$\varepsilon = \frac{\delta L}{L}$$

Hook's law

$\sigma \propto \varepsilon$ (with in the proportional limit) (No elastic limit).
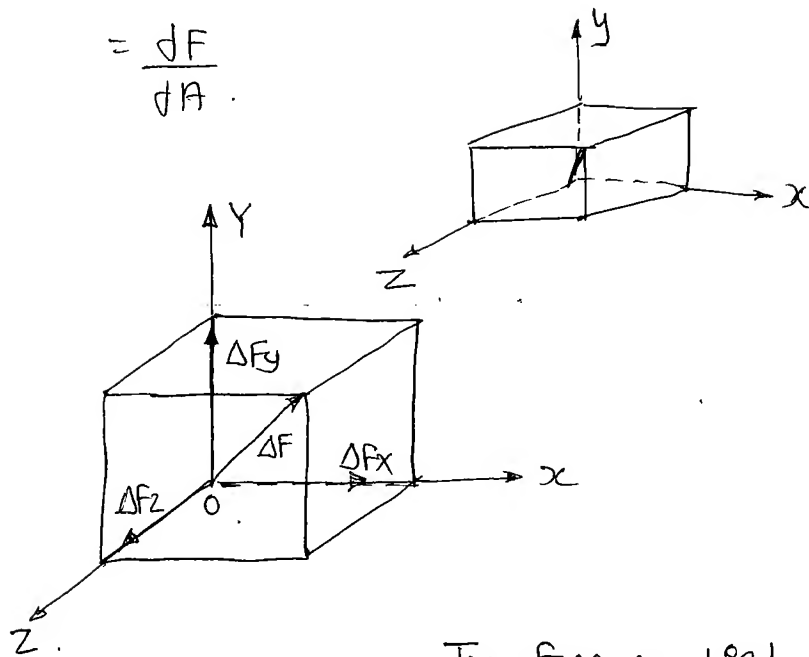
Stiffness (K)

$$k = \frac{load}{deflection}$$

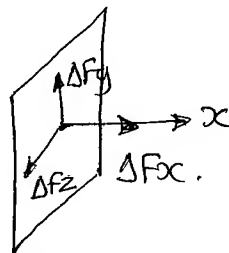$$= \frac{F}{\frac{Fl}{AE}} = \frac{AE}{L}$$

Stress at a point.

$$\sigma_t = \underset{\Delta A \to 0}{Lt} \frac{\Delta F}{\Delta A}$$

$$= \frac{dF}{dA}.$$

→ Force ⊥^{er} to x axis

Two face are ⊥^{er} to x axis.

Tensor
(vector qunt) $\sigma_{xx} = \underset{\Delta A \to 0}{Lim} \frac{\Delta Fx}{\Delta A}.$

Force Normal   Force
to co-oordinate   direction

$$\sigma_{xy} = \underset{\Delta A \to 0}{Lim} \frac{\Delta Fy}{\Delta A}.$$

$$\sigma_{xz} = \underset{\Delta A \to 0}{lim} \frac{\Delta Fz}{\Delta A}.$$

→ Force ⊥^{er} to Y axis (same).

  "   "   " Z axis   "

Stress tensor

Direct stress
Normal stresses (volume change)

Distorsion
Stress

$$\sigma = \begin{array}{c} x \\ y \\ z \end{array} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Normal stress — It changes the size (volume changes)
It is called as direct stress (No shape change)

Distortion stress

σ_{xx}

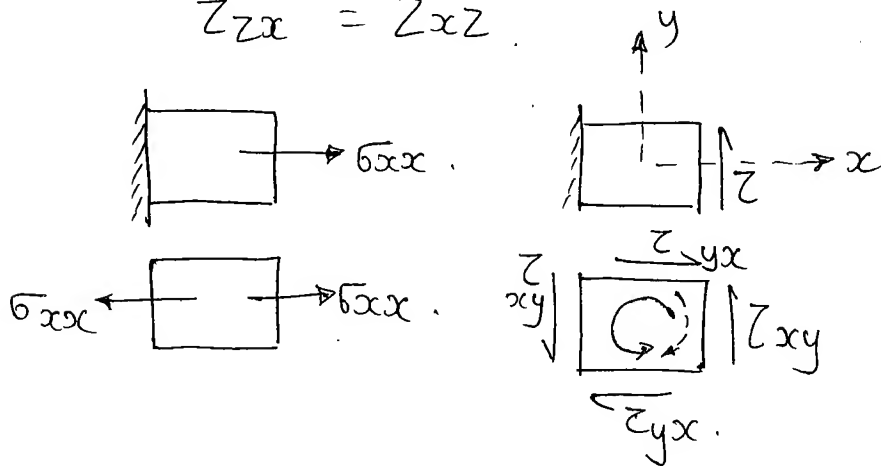σ_{ft}
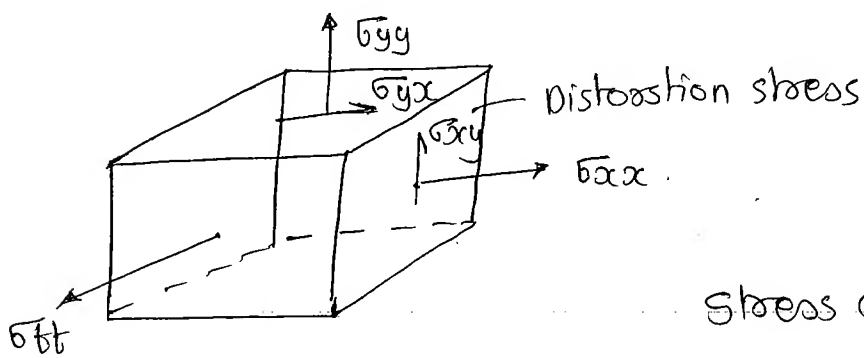
Stress component-g.

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

Complementary shear stress are

$\tau_{xy} = \tau_{yx}$

$\tau_{yz} = \tau_{zy}$

$\tau_{zx} = \tau_{xz}$

6 stress components are req to define the total stress at a point in the space.

σ_{xx} τ_{xy}
σ_{yy} τ_{yz}
σ_{zz} τ_{zx}

Two dimention stress. (Plane stress)

Ex:- ① Thin sheets (laminar)

 x (thickness is very less)

② Thin cylindrical shell

σ_b, σ_l and p (pressure) is neglected.

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$$

3 stress compents are required to define the stress

$\sigma_{xx}, \sigma_{yy}, \tau_{xy} = \tau_{yx}$.

Principle Stress.

Principle plain – A plain carrying zero ~~principle~~ shear stress.
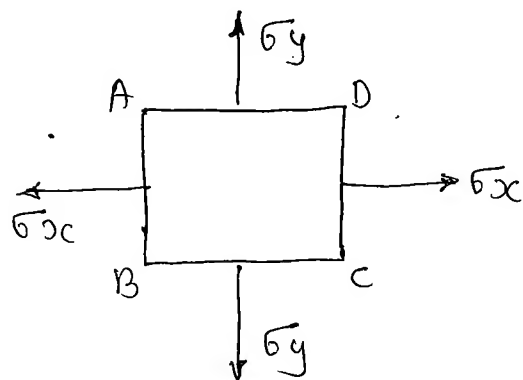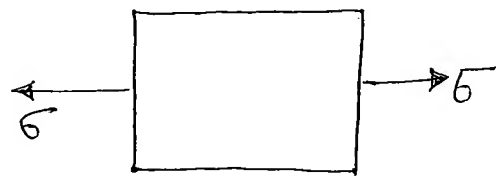
Principle stress – The stress (normal) acting on the principle plain.

Principle palanes and stresses.

Principle plane.

zero shear stress $\begin{cases} AB \text{ or } CD & \sigma_1 = \sigma \\ AD \text{ or } BC & \sigma_2 = 0. \end{cases}$

Principle plane.

AB or CD $\sigma_1 = \sigma_y$ Major

AD or BC $\sigma_2 = \sigma_x$ Minor

$\sigma_y > \sigma_x$.

Biaxial type

$\longrightarrow$

AB or CD $\sigma_2 = \sigma_x$

AD or BC $\sigma_1 = \sigma_y$

$\sigma_y > \sigma_x$

$\longrightarrow$

$\sigma_y > \sigma_x$
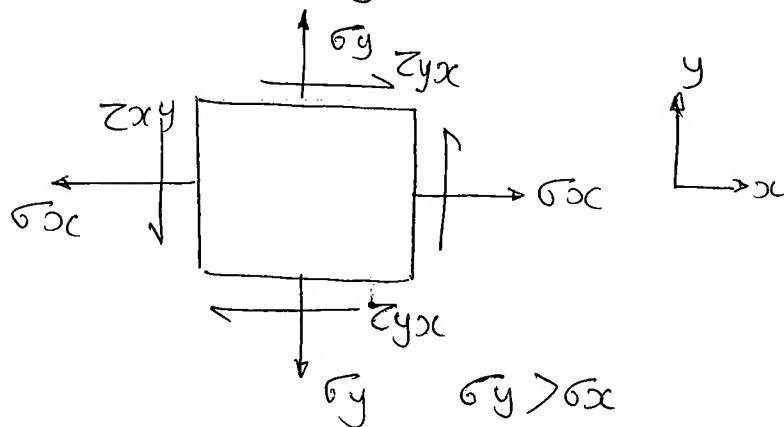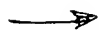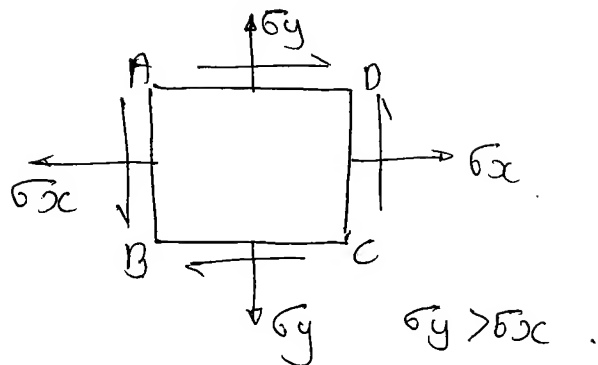
Principle ~~planes~~ stress

$$\sigma_1 \,\&\, \sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\frac{(\sigma_x - \sigma_y)^2}{2} + \tau_{xy}^2}$$

Principal planes

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$\longrightarrow$

$\sigma_y > \sigma_x$

$$\sigma\theta = \left(\frac{\sigma x + \sigma y}{2}\right) + \left(\frac{\sigma x - \sigma y}{2}\right)\cos 2\theta + \tau xy \sin 2\theta$$
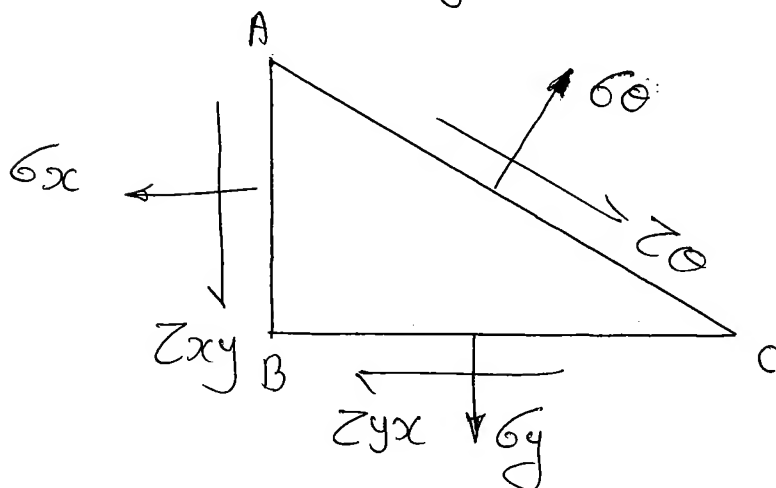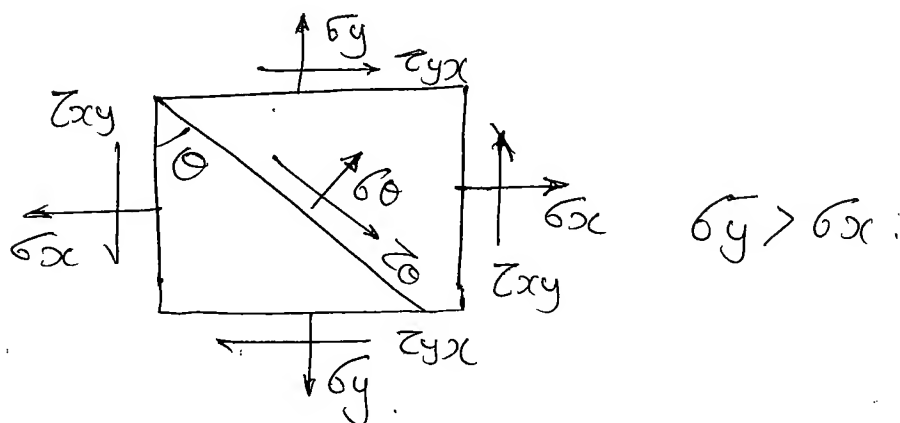
$$\tau\theta = \left(\frac{\sigma x - \sigma y}{2}\right)\sin 2\theta - \tau xy \cos 2\theta.$$

downward shear stress
positive direction

Mohr's Circle.

$$\sigma = \begin{bmatrix} \sigma x & \tau xy \\ \sigma yx & \sigma y \end{bmatrix}$$

Assume
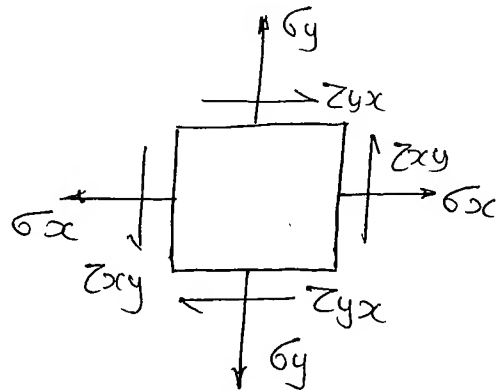$$\sigma x > \sigma y.$$

$OA = \sigma x$

$OB = \sigma y$.

$AD = -\tau xy$

$BE = +\tau yx$.

Sign Convention

→ Normal stress – abscissa
   Tensile - - - + ve
   Compressive - - -ve

→ Shear stress —ordinate
   Clockwise + ve
   c.c.w   -ve
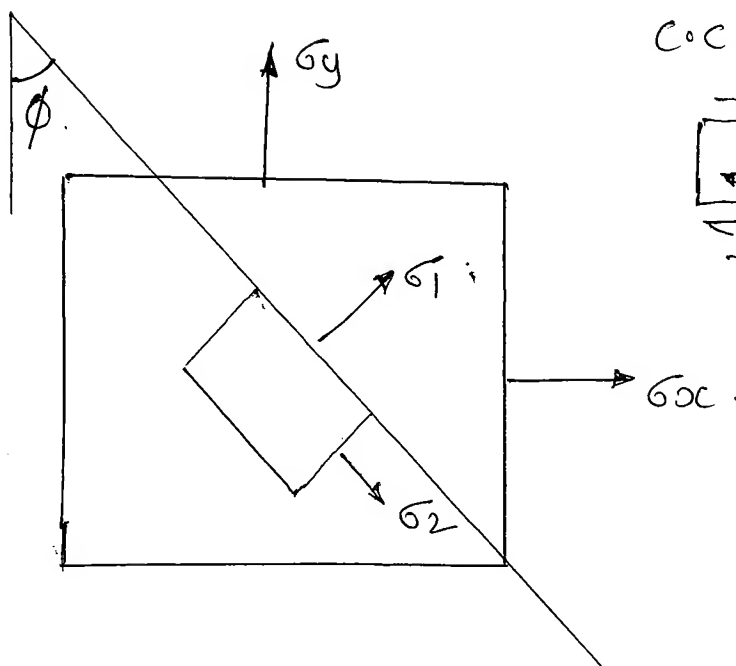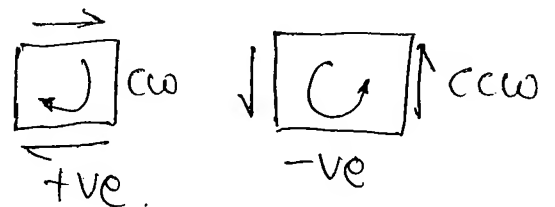
2ϕ - hat twise the actual value.
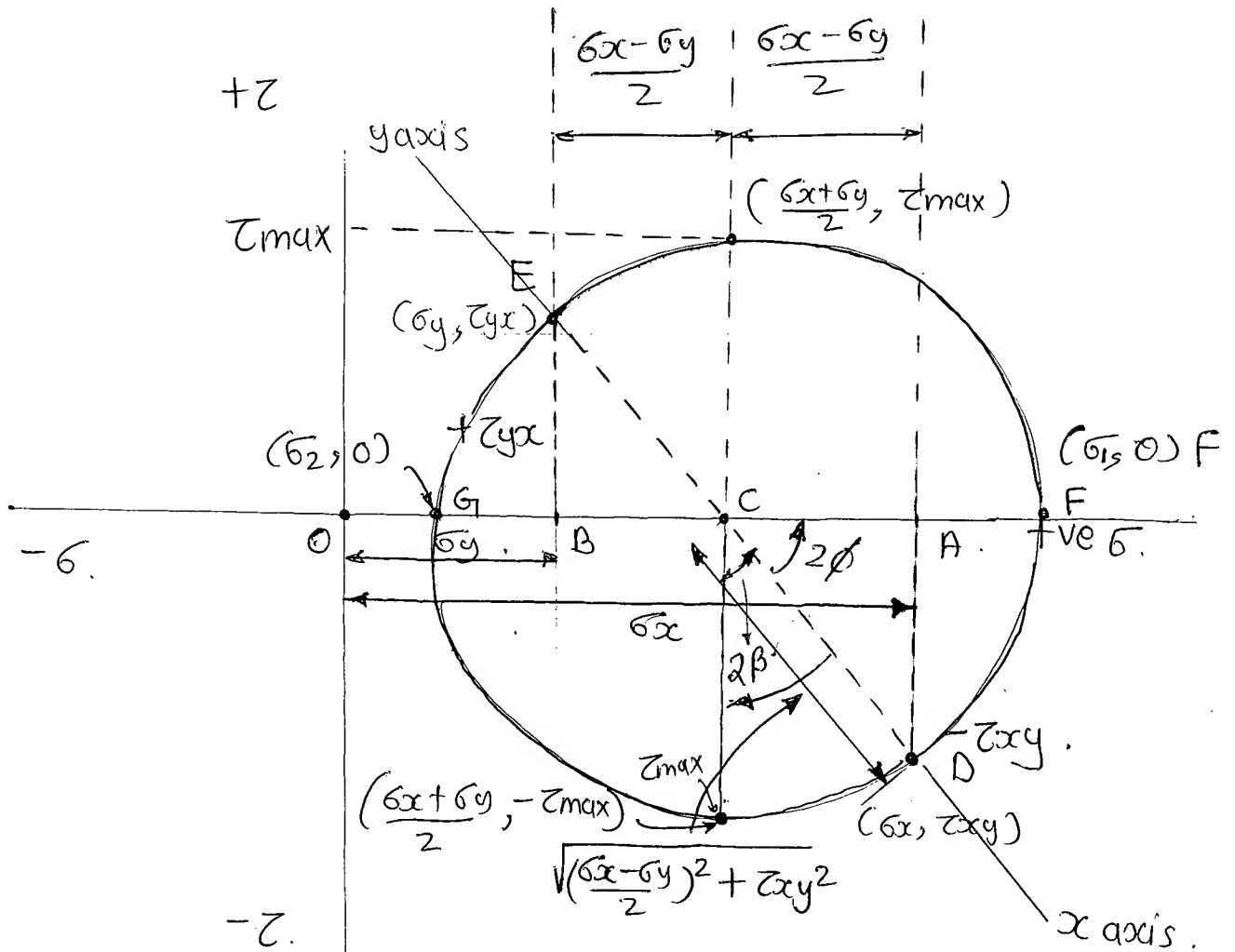
$\sigma_1 = OF$ — Maximum stress with zero shear stress. Hence Max shear principle stress.

$\sigma_2 = OG$ — Min stress with zero shear stress. Hence Min principle stress.

$$AC = BC = \frac{AB}{2} = \frac{\sigma x - \sigma y}{2}$$

$$AB = \sigma x - \sigma y$$

$$OC = \sigma y + \frac{\sigma x - \sigma y}{2} = \frac{\sigma x + \sigma y}{2} \quad (OB + OC)$$

$$OF = \sigma_1 = OC + CF \qquad (CF = CD = R)$$
$$= OC + CD.$$

$$\sigma_1 = \frac{\sigma x + \sigma y}{2} + \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 \quad - \quad \sigma x + \sigma y - R.$$

$$\tau_{max} = \text{Radius of mohr's circle} = R.$$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \tau_{max}.$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \tau_{max}.$$

ORIENTATION OF PRINCIPAL PLANES.
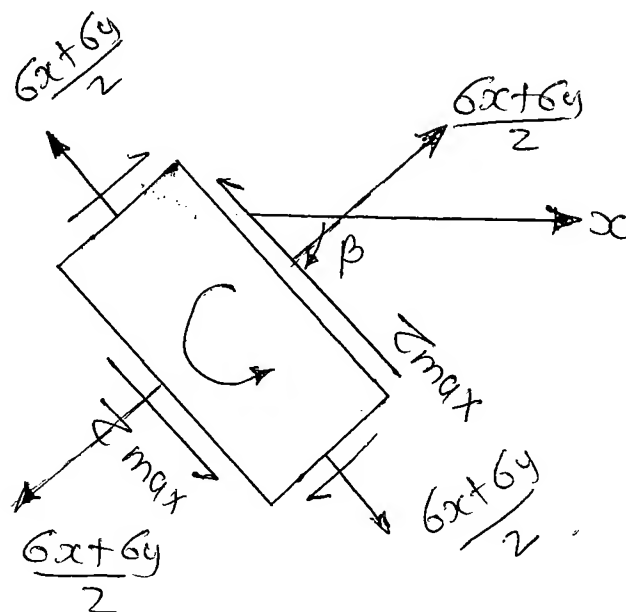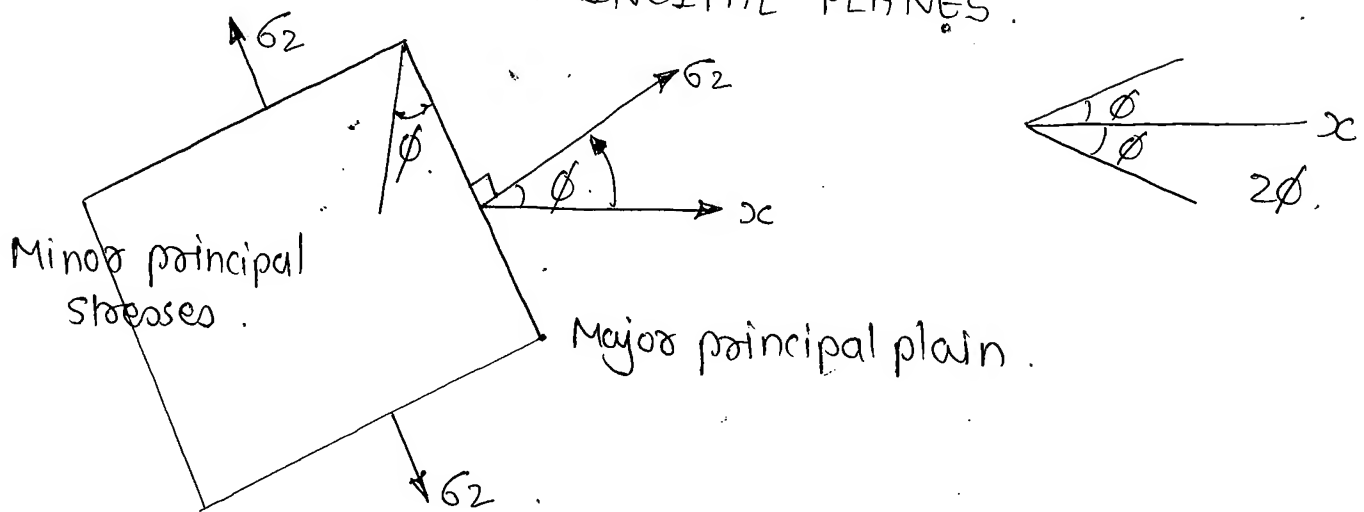

Minor principal stresses.

Major principal plain.

Design for static strength.

Q1] The strength of stress at a point is given, in Mpa is

$$\sigma = \begin{bmatrix} 40 & 0 \\ 0 & -30 \end{bmatrix}$$ The maximum shear stress.

a] 20 MPa b] 30 MPa c] 35 ✓ d] 40 MPa.

$\sigma_1 = 40$ $\sigma_2 = -30$. $\sigma_3 = 0$

$$\tau_{max} = \text{max of} \begin{cases} \left|\dfrac{\sigma_1 - \sigma_2}{2}\right| \\ \left|\dfrac{\sigma_1 - \sigma_3}{2}\right| \\ \left|\dfrac{\sigma_2 - \sigma_3}{2}\right| \end{cases} = \begin{cases} \dfrac{40 - (-30)}{2} \\ \pm\dfrac{40}{2} \\ \dfrac{-30}{2} \end{cases} \quad \begin{array}{l} \dfrac{70}{2} = 35 \\ \dfrac{40}{2} = 20 \\ \dfrac{-30}{2} = 15 \end{array}$$

$$\sigma = \begin{bmatrix} 40 & 0 \\ 0 & 30 \end{bmatrix} \qquad \tau_{max} = \dfrac{40-30}{2} = \dfrac{10}{2} = 5$$

$$\dfrac{40}{2} = 20 \checkmark$$

$$\dfrac{30}{2} = 15$$

2] The stress at a point in MPa is. $\sigma = \begin{bmatrix} 50 & 10 \\ 10 & 30 \end{bmatrix}$

The principle stress.

$$\sigma = \begin{bmatrix} 50 & 10 \\ 10 & 30 \end{bmatrix} \qquad \sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1 = 50$ $\sigma_2 = 30$ $\sigma_3 = 0$ Principal stress.

$$\sigma_A = \dfrac{50 + 30}{2} + \sqrt{\left(\dfrac{50 - 30}{2}\right)^2 + 10^2} = 54.1421\,\text{MPa},$$

$$\sigma_B = \dfrac{50 + 0}{2} + \sqrt{\left(\dfrac{50}{2}\right)^2 + 10^2} = 51.92\,\text{MPa}.$$

$$\sigma_C = \dfrac{30}{2} + \sqrt{\left(\dfrac{30}{2}\right)^2 + 10^2} = 33.02\,\text{MPa}$$

Shear stress

$$\sigma_A = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = 14.14$$

$$\sigma_B = 26.92\,\text{MPa}. \checkmark$$

③ The stress tensor at a a point is given by by (in MPa)
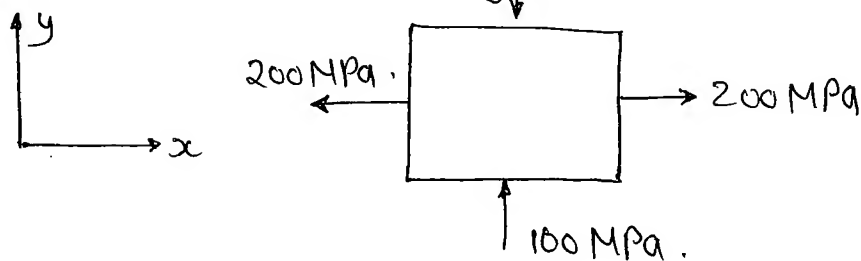
$$T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -20 \end{bmatrix}$$

The principal stresses are

$$T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -20 \end{bmatrix}$$ $\sigma_3 = -20 \, MPa$.

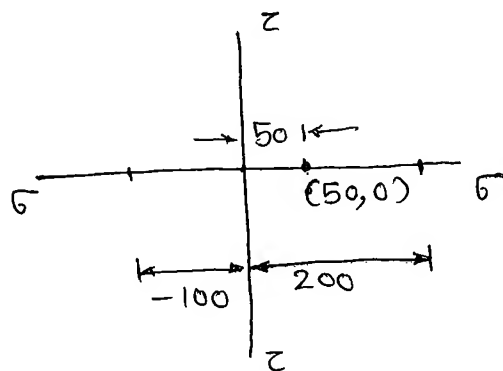$$\sigma_1 = \frac{10+20}{2} + \sqrt{\left(\frac{10+20}{2}\right)^2 + 5^2} = 22.071$$

$$\sigma_2 = \frac{10+20}{2} \pm \sqrt{\left(\frac{10+20}{2}\right)^2 + 5^2} = 7.93 \, MPa.$$

④ Consider a two dimention state of stress given form element as shown in Fig.

What are the co-ordinate centre of mohr's circle .

a) (0,0)   b) (100,200)   c) (200,100)   d) (50,0) ✓

⑤ In a strained material one of the principal stresses is twise the other. The maximum shear stresses in the same cuse is τ_{max}, then what is the max value of principal stress

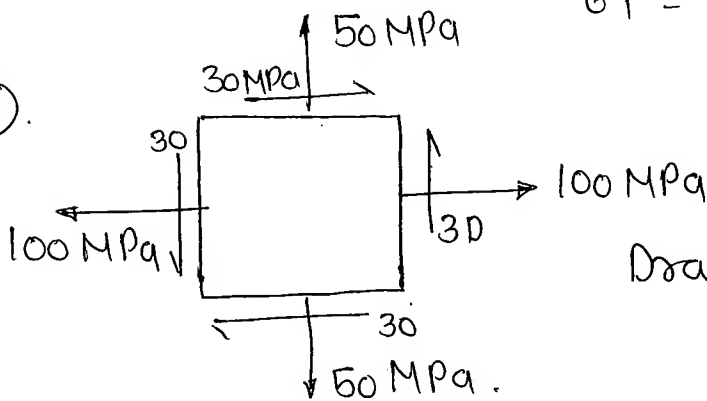a] τ_{max} b] $2\tau_{max}$ ✓   c] $4\tau_{max}$ d] $8\tau_{max}$.

$$\sigma_1 = 2\sigma_2$$

$$\tau_{max} = \frac{2\sigma_1 - \sigma_2}{2} = \frac{2\sigma_2 - \sigma_2}{2}$$
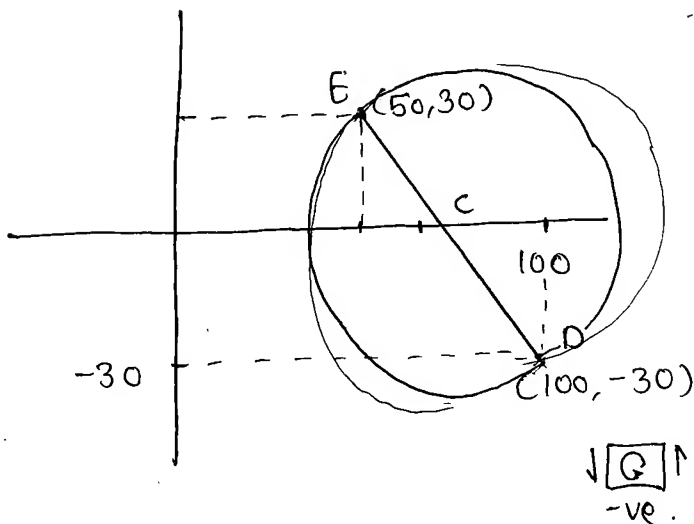
$$\sigma_2 = 2\tau_{max}$$

$$\sigma_1 = 4\tau_{max} ✓$$

⑥.

50 MPa
30MPa
30
100 MPa
30
100 MPa
30
50 MPa.

Draw the mohr's circle.

$1cm = 10 MPa$

E (50,30)
C
100
-30
D
(100,-30)
↓ Q ↑
-ve.

$$\tau_{max} = \max \text{ of } \begin{cases} \dfrac{\sigma_1 - \sigma_2}{2} \\[4pt] \dfrac{\sigma_1}{2} \\[4pt] \dfrac{\sigma_2}{2} \end{cases}$$

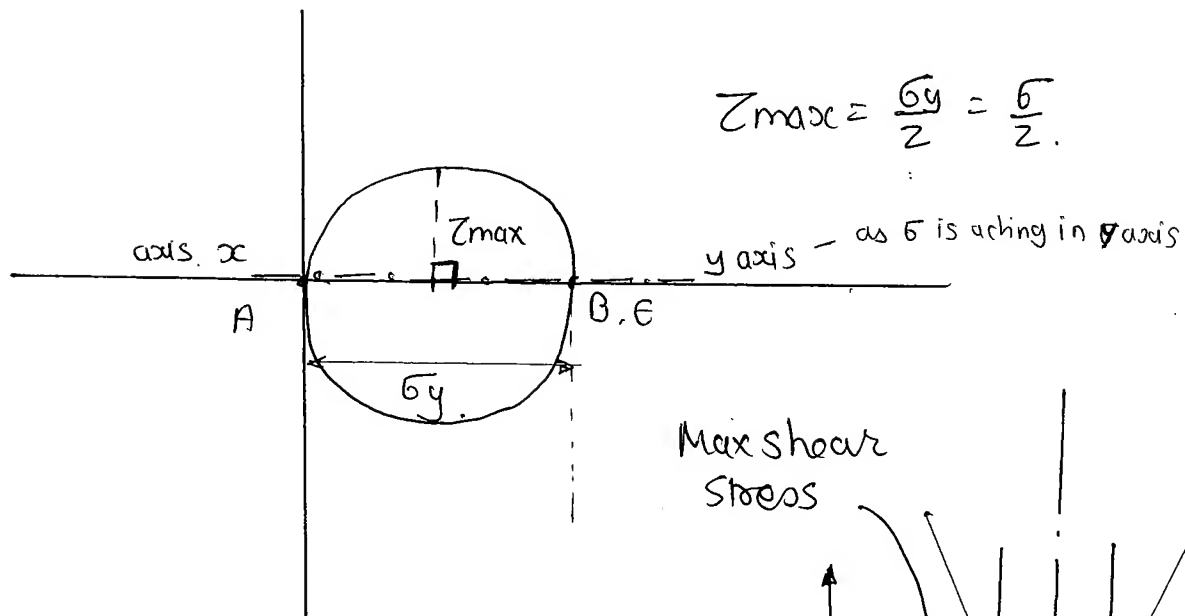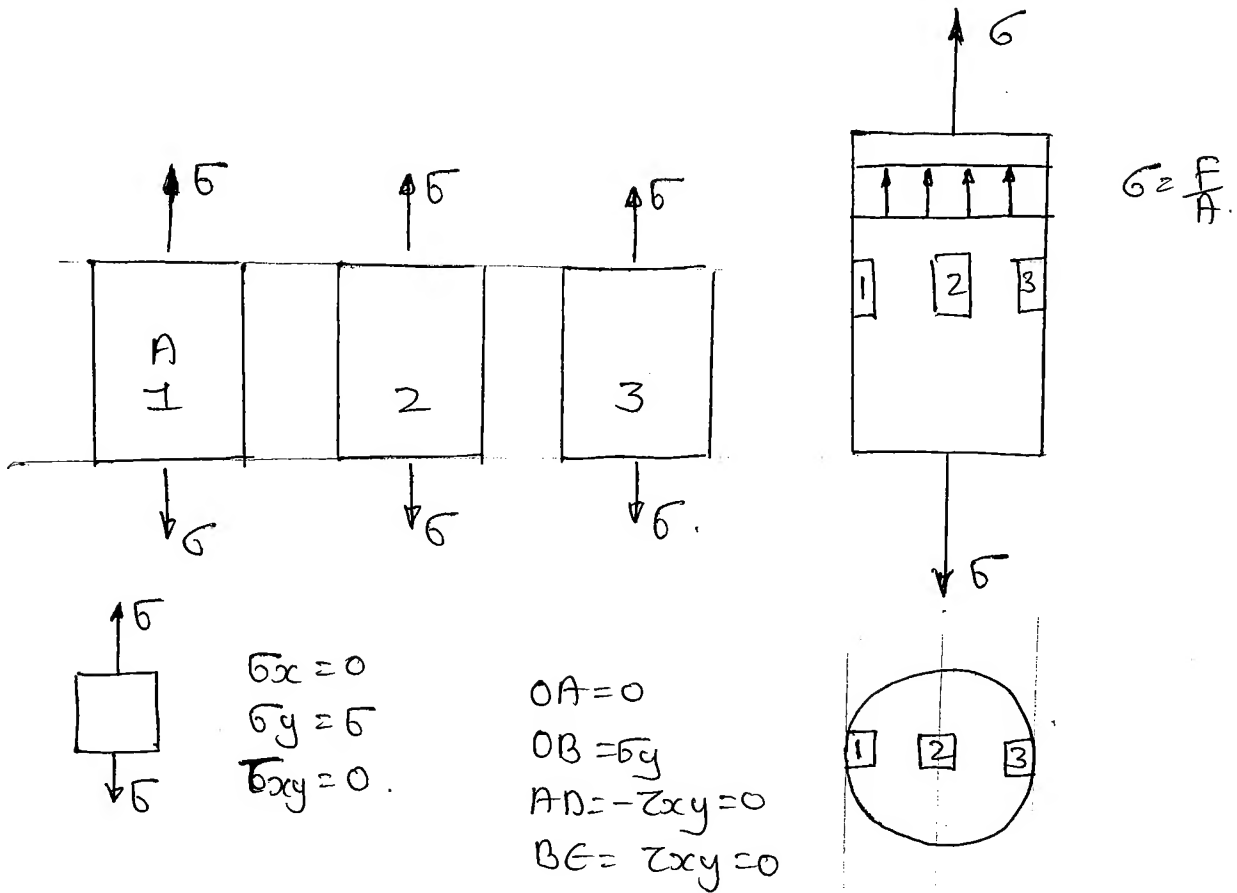$$= \frac{2\sigma_2 - \sigma_2}{2} = \frac{\sigma_2}{2}$$

$$= \frac{2\sigma_2}{2} \qquad = \sigma_2 = \tau_{max}.$$

$$= \frac{\sigma_2}{2}$$

$$\sigma_1 = 2\tau_{max}$$

⑦. Draw a mohr's circle under uniaxial state of stress.

Mohr's circle for simple tension

$6 = \frac{F}{A}$

$6x = 0$
$6y = 6$
$\tau xy = 0$.

$OA = 0$
$OB = 6y$
$AD = -\tau xy = 0$
$BE = \tau xy = 0$

$\tau max = \frac{6y}{2} = \frac{6}{2}$.

axis x τmax y axis — as 6 is acting in y axis

A   B.E

$6y$.

Max shear
stress

45° 45°

Cup &
Cone
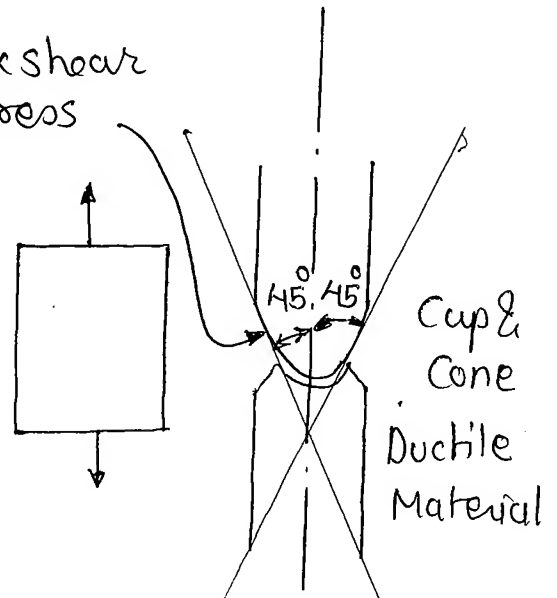
Ductile
Material

Mohr's Circle For Torsion stresses.

$$\frac{T}{J} = \frac{Z}{R}.$$

$$Z \propto R.$$

$$Z\uparrow \quad R\uparrow$$
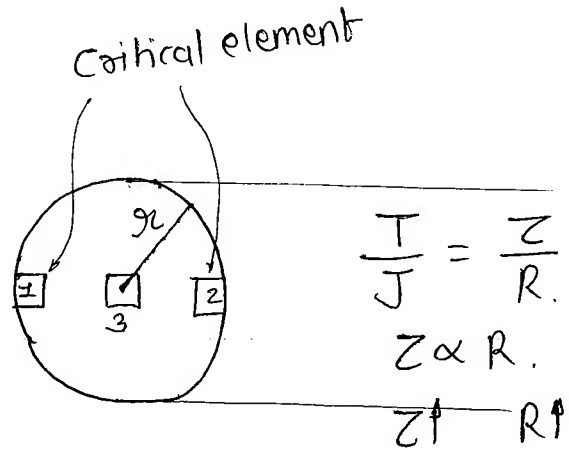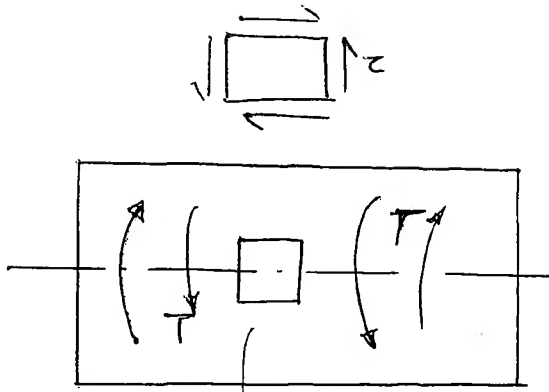
$$R=0 \quad Z=0.$$

Direction of shear stress is depends upon the direction of torque we applied.

Pure stress
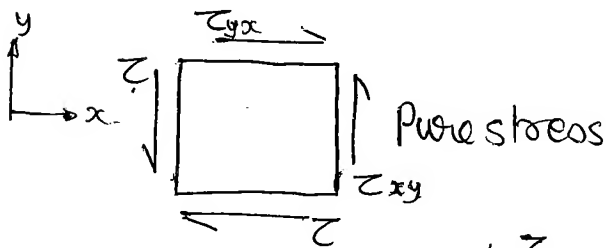
Z_{xy} -ve
Z_{yx} +ve

Brittle material — Seperation of layer

Ductile   ''   — Sliding

MS.

Shear failure under torsion
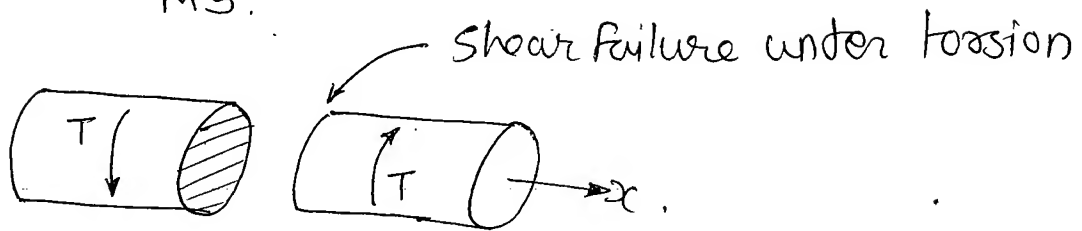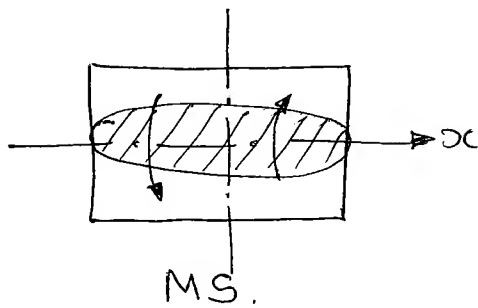
Resistance is less. Hence failure take place at a plane $\perp$ to x axis.

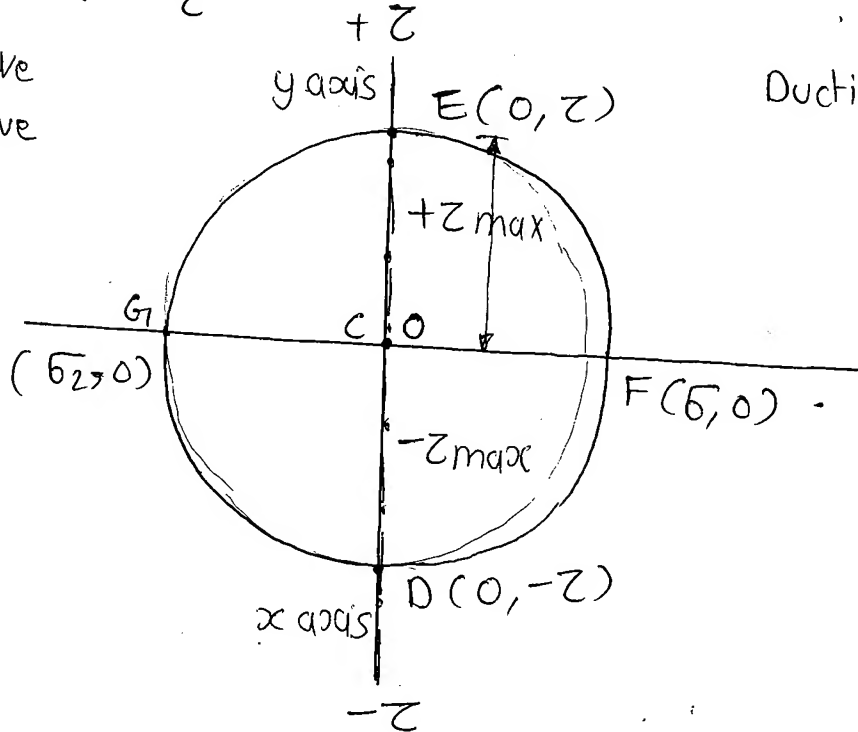σ

CI

Under Normal stress
Brittle material

σ

Fractured.
Separated.

σ

σ
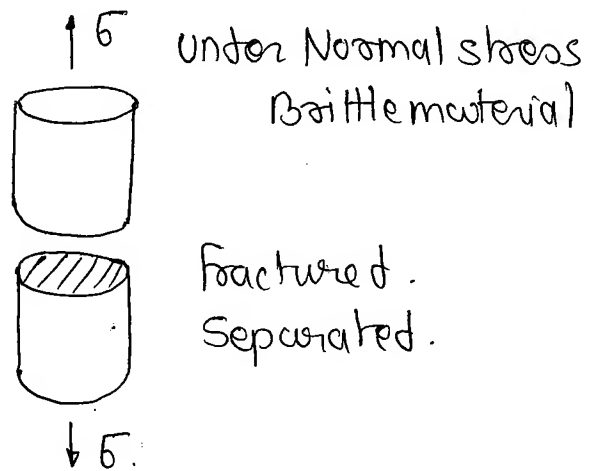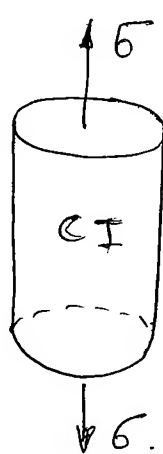
Plane of max Normal stress at an angle of $45°$
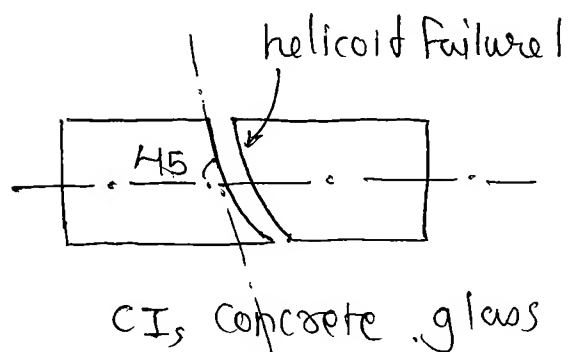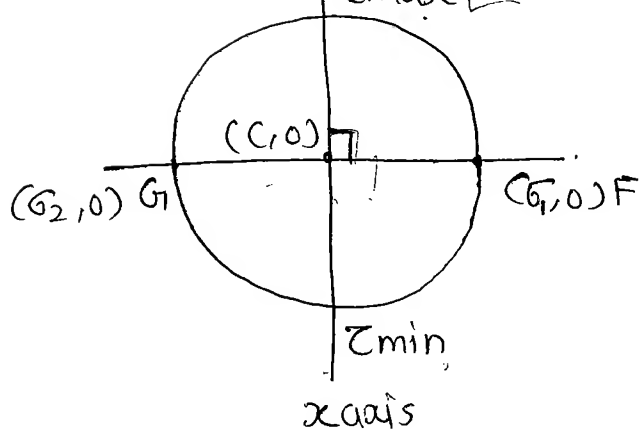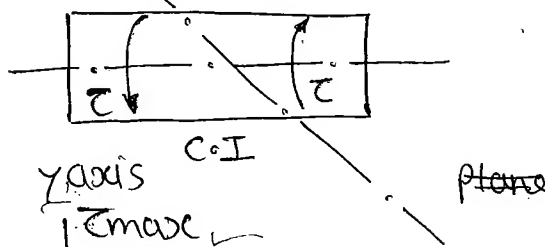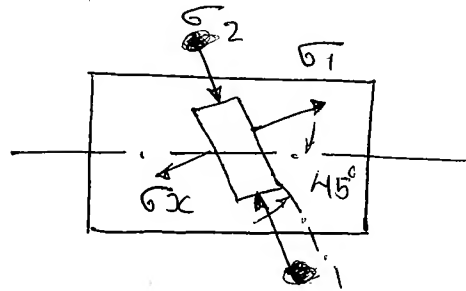
τ CI τ

Plane

y axis

τmax

$(C,0)$

$(\sigma_2,0)$ G $(\sigma_1,0)$ F

τmin

x axis

helicoid Failure!

$45°$

CI, Concrete, glass
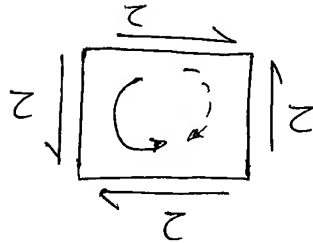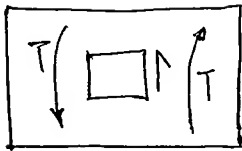
Mohr's circle for Hydrostatic stress.

The stress is same in all direction. in compression or in tension.

Compression

tension.

D $(-\sigma, 0)$
E $(-\sigma, 0)$

$(-\sigma, 0)$

$-\sigma$ D, E .

$-\sigma$
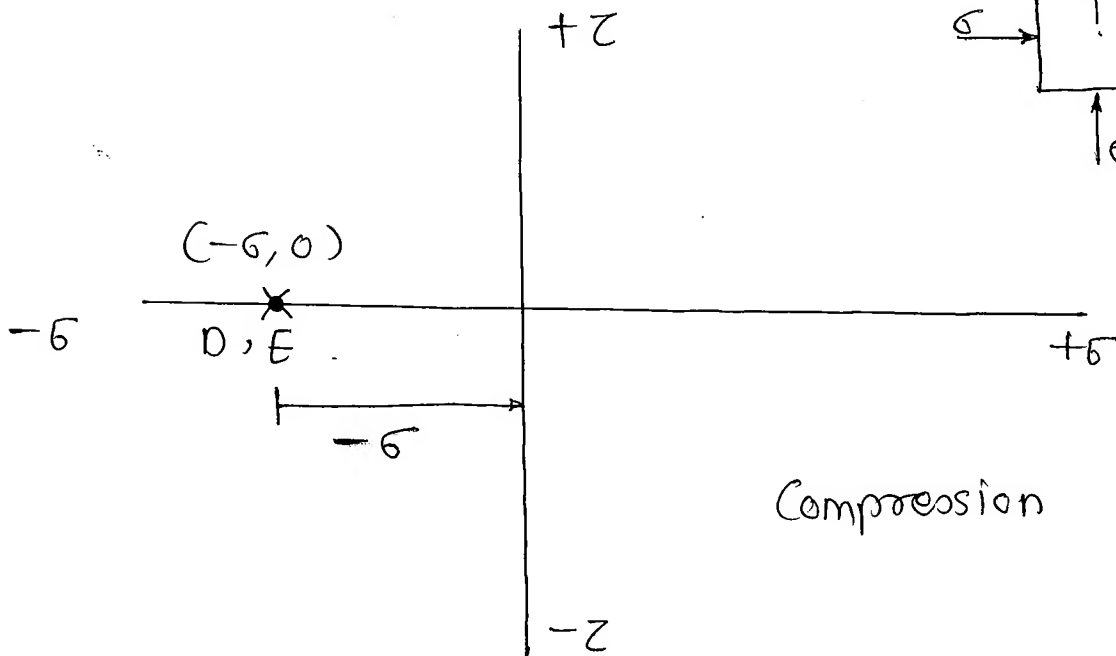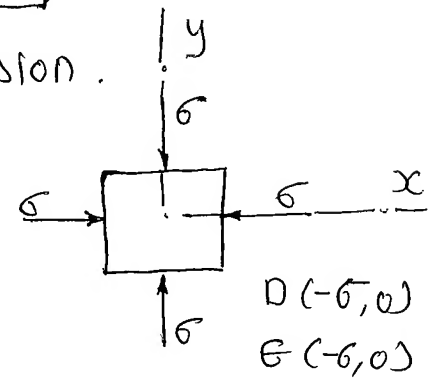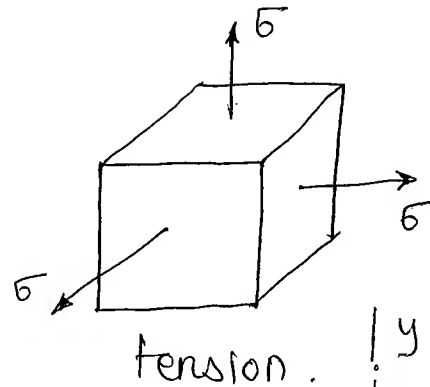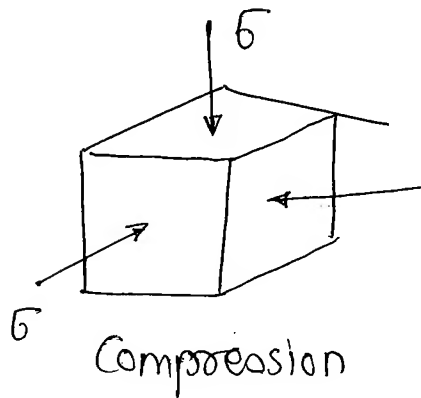
Compression

A point -ve site of absisa is called as hydrostatic stress under compression
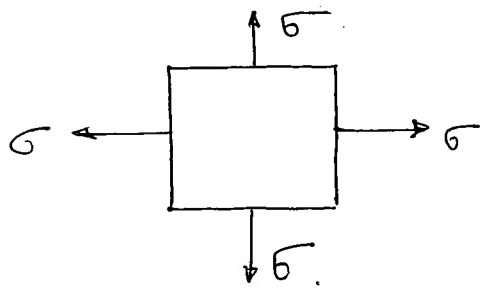
$D(\sigma, 0)$ – x plane
$E(\sigma, 0)$ – y plane

2 axis

D, E

x axis σ

$-\sigma$
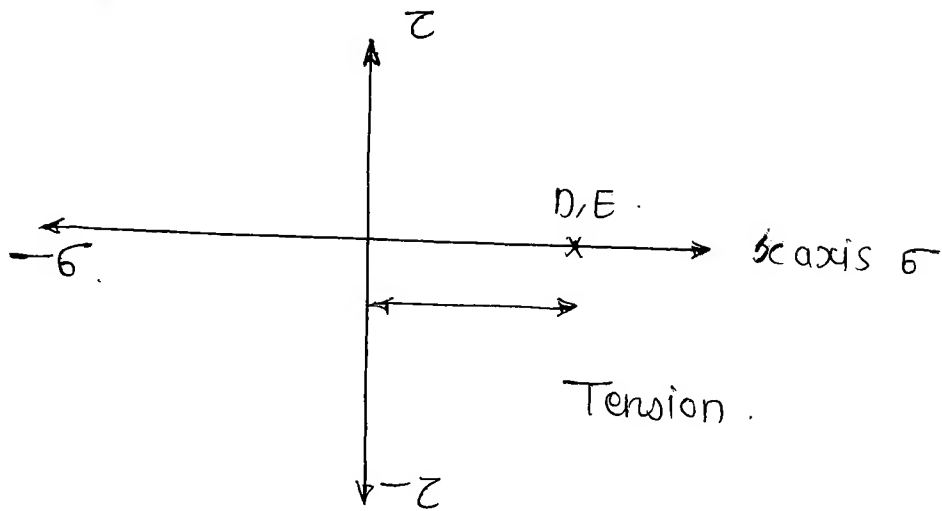
Tension.

A point on the +ve side of the x axis is called as Hydrostatic stress under tension.

	Ductile Material	Brittle Material.
Failure mode	yielding (slipping of plane under shear)	Separation or fracture.
Strengths	$S_{yt} = S_{yc}$ Strength at yield point under tension or comp.	~~S_{yt}~~ = ~~S_{yc}~~ $S_{ut} \neq S_{uc}$ (Not isotropic)

σ

S_{ut} - - - - - - -
S_{yt} - - - - - Ductile
S_{ut} - - - Brittle

Strain.

(Isotropic)
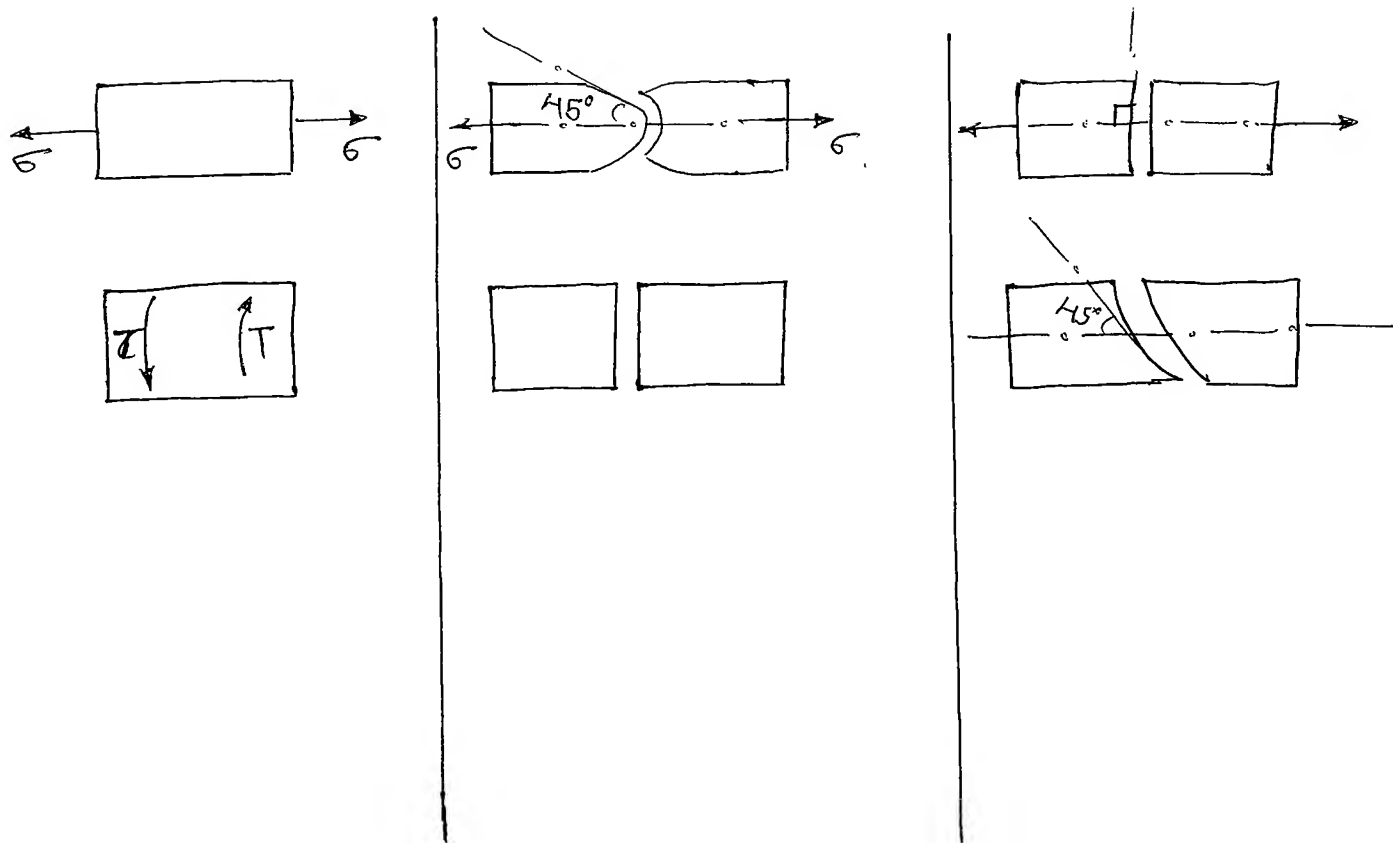
Theories of Failure.
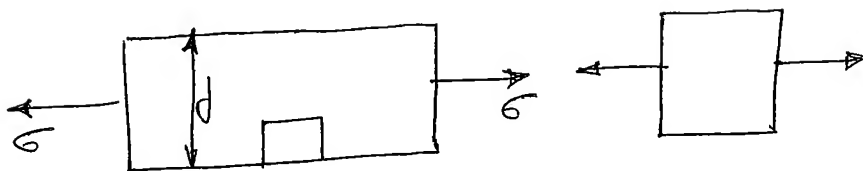
$$\sigma = \frac{F}{\frac{\pi}{4}d^2} = strength.$$

The component may get fail because of $S_{ut}, S_{uc}, S_{yt}, S_z$ and the max value out of this is consider.

Establishes a relationship between principal stresses. together with some elastic constants and the strength of the component determine by simple torsion.

E, G or C or N, K, μ — cont.

List 1	List 2	Theories of Failure
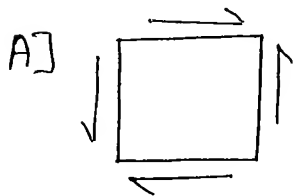
A] Combine bending & torsion.

	A	B	C	D
a)	2	1	3	4

B] Torsion.

b)	3	4	2	1
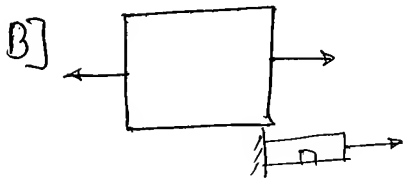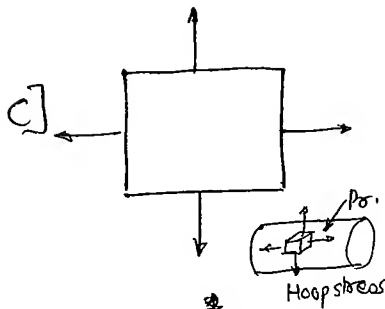
C] Thin cylindrical Shells under pressure

Po.

Hoop stress

c)	2	4	3	1
✓
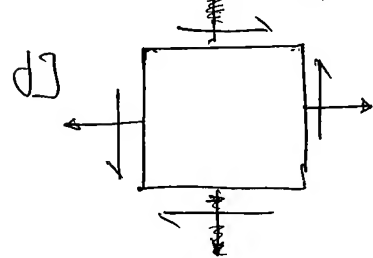
d] Tie bar subjected to tensile force

d)	3	1	2	4 .

M ⟩ M

M   6   6

Q1] The maximum shear stress in a bar of c/s are A mm² and subjected to a tensile load of p newtons is

a] 0   b] P/A   c] ✓P/2A   d] P/4A.

$T_{max} = \sigma/2$
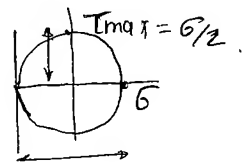
Q2. A steel saw blade 1mm thick is bent into an arc of circle of 500 mm radius. The width of the blade is 15mm and E = 210 GPa.

1. Fluxtural stress induced is.

a] $\frac{P}{110}$ b] $\frac{P}{210}$ c] $\frac{P}{250}$ d] $\frac{P}{350}$
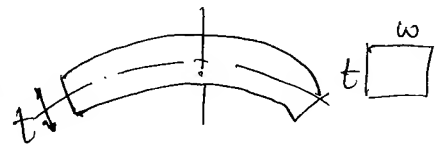
$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}.$$

$$\frac{\sigma_b}{y} = \frac{E}{R}.$$

$$\sigma_b = E \times \frac{y}{R} = 210 \times 10^3 \times \frac{t}{2} \times \frac{1}{500}$$

$$\sigma_b = 210 t \cdot \quad t=1.$$

$$\boxed{\sigma_b = 210}$$

Theory of Failures.

① Rankine's theory (or) Maximum normal stress theory.

If $\sigma_1 > \sigma_2 > \sigma_3$.

then σ_1 = Max normal stress
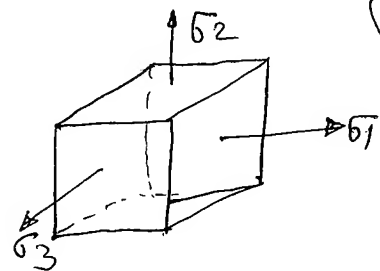
If $\sigma_1 > S_{yt}$ - duchile. $\Big\}$ Component of Failure.
$\sigma_2 > S_{ut}$ - brittle.

$\sigma_1 \leq S_{ut}$ - safe
$\sigma_1 \leq S_{ut}$ - safe.

Design Equation.

$$\sigma_1 = \frac{S_{yt}}{F \cdot S} \quad - \text{ Duchile} \qquad F \cdot S = \text{factor of Safety.}$$

$$\sigma_2 = \frac{S_{ut}}{F \cdot S} \quad - \text{ Brittle}.$$

Validities — Valid for brittle material and not valid for
               : ductile material.
        - Disregards the effect of other stresses.

$\sigma_2 = 50\,MPa$.

$\sigma_1 = 100\,MPa$.

$\sigma_3 = 99.9\,mpa$.

Graph.

σ_2

$\sigma_1 \longleftarrow$ $\longrightarrow \sigma_1$

σ_2.
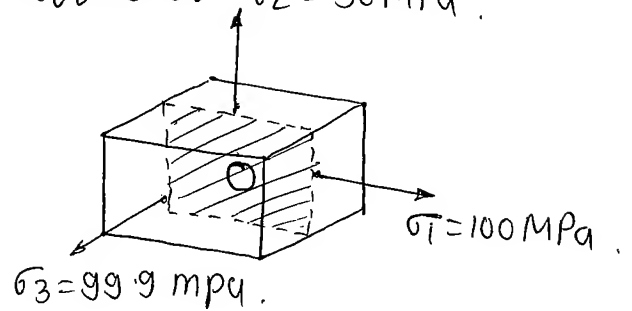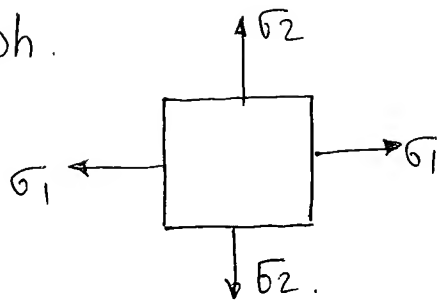
a] If $\sigma_1 > \sigma_2$

$$\sigma_1 = \frac{Syt\,(Ductile)}{F.S = 1}.$$

$\sigma_1 = Syt$.

$x = a$.

b] If $\sigma_2 > \sigma_1$.

$\sigma_2 = Syt$

σ_2

$\sigma_2 \longrightarrow$ $\longleftarrow \sigma_1$

σ_2
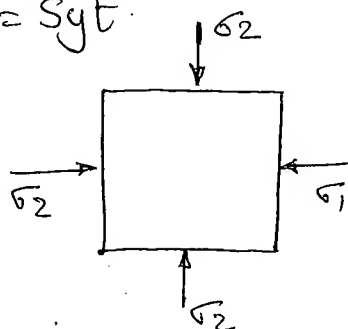
c] If $\sigma_1 > \sigma_2$

$\sigma_1 = Sye = -Syt$ (isotropic material)

d] If $\sigma_2 > \sigma_1$

$\sigma_2 = -Syt$

For Ductile material

$\sigma_2 = Syt$

$+\sigma_2$

$\sigma_1 = Syt$

B   Syt   A

yielding   Safe $2syt$

$-\sigma_1$ $-syt$ Zone syt $+\sigma_1$

$\sigma_1 = -Syt$

C $-Syt$ D

$-\sigma_2$

Square.   Elastic

plastic or
Failure.
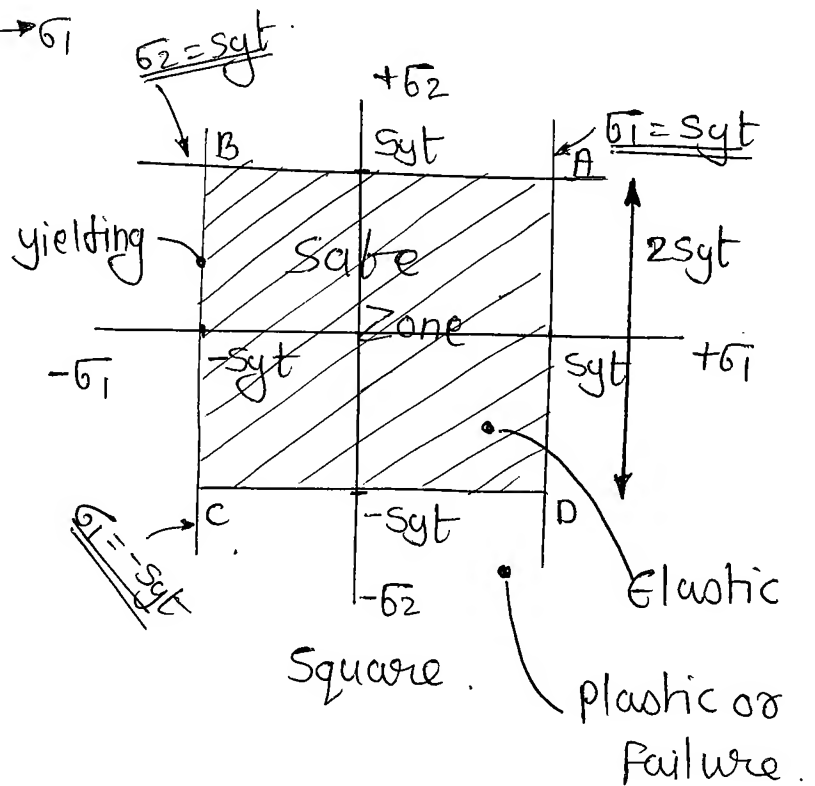
Graph — Brittle Material.

a) If $\sigma_1 > \sigma_2$
$$\sigma_1 = Sut$$
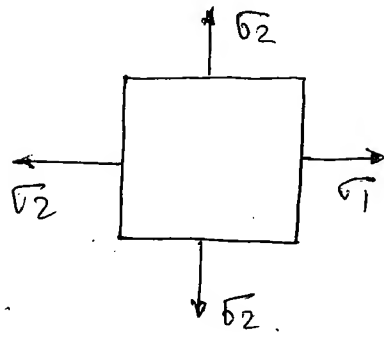
b) If $\sigma_2 > \sigma_1$
$$\sigma_2 = Sut$$
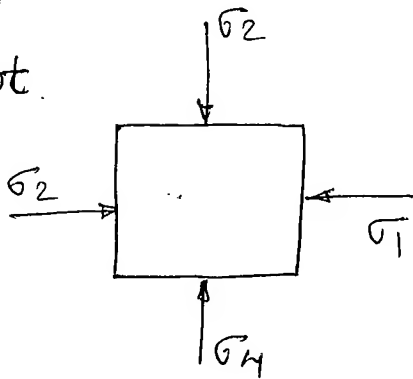
c) If $\sigma_1 > \sigma_2$
$$\sigma_1 = Suc$$

d) If $\sigma_2 > \sigma_1$
$$\sigma_2 = Suc$$
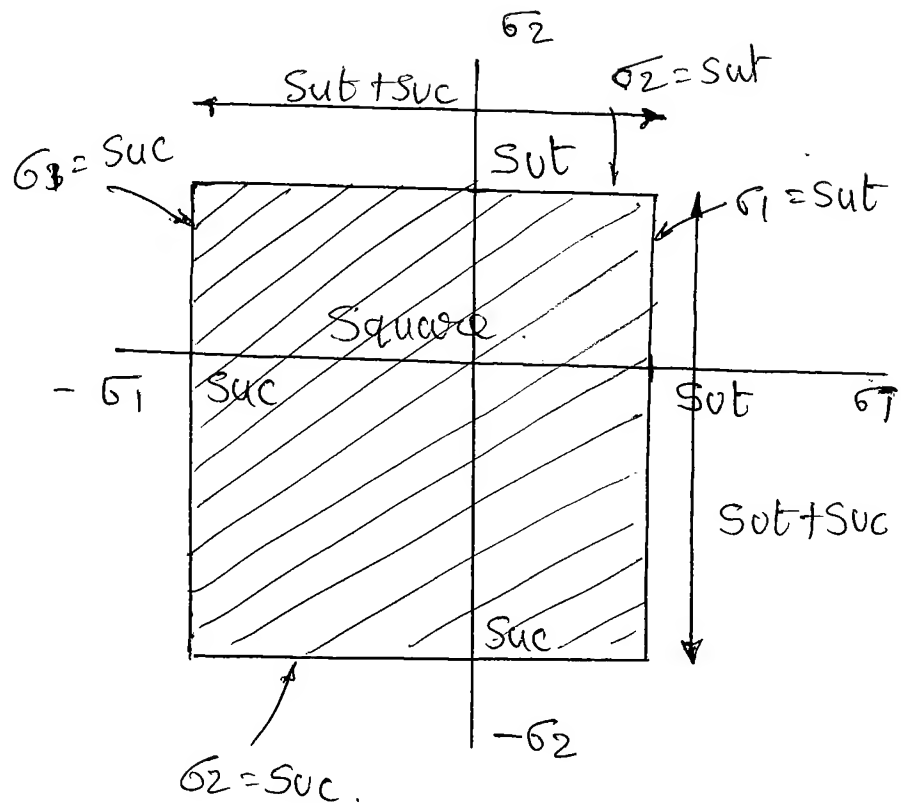
A component subjected to complex state of stress is said to be faild when the maximum normal stress at a point in the material exceeds the strength at yeild point under simple tension.

② Guest's theory or Tresca's theory or Maximum shear stress theory.

Maximum shear stress.

$$\tau_{max} > S_{sy} - \text{Failed.}$$

S_{sy} = shear strength at yield point.

Design

$$\tau_{max} = \frac{S_{sy}}{F.S}$$

If Maximum shear stress

$$\tau_{max} < S_{sy}. \quad - \text{safe.}$$

$$\tau_{max} = \text{Max of} \begin{cases} \frac{\sigma_1 - \sigma_2}{2} \\ \frac{\sigma_1 - \sigma_3}{2} \\ \frac{\sigma_2 - \sigma_3}{2} \end{cases} = \text{max of} \begin{cases} \left|\frac{\sigma_1 - \sigma_2}{2}\right| \\ \left|\frac{\sigma_1}{2}\right| \\ \left|\frac{\sigma_2}{2}\right| \end{cases}$$

If $\sigma_1 > \sigma_2 > \sigma_3$ and all are like stresses then,

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{S_{yt}}{2 \times F.S}$$

$$\boxed{\sigma_1 - \sigma_3 = \frac{S_{yt}}{F.S}}$$

$$\tau_{max} = \frac{\sigma}{2}$$

$$S_{sy} = \frac{S_{yt}}{2}$$

$$\tau_{max} = \frac{S_{st}}{F.S}$$
$$= \frac{S_{st}}{2 \times F_S}$$

* Validity

① Valid for only ductile material
② Not valid for brittle material
③ Not explained the failure phenomenon under hydrostatic stress

τ_{max} fmln

Graph. (Ductile material)

①

σ_2 up, σ_1 right, σ_2 down (element diagram)

Graph plot with axes σ_2 and σ_1:
- $\sigma_2 - \sigma_1 = syt$
- $\sigma_1 = -syt$
- $\sigma_2 = syt$
- syt
- $C (0, syt)$
- $\sigma_1 = syt$
- $\sigma_1 - \sigma_2 = syt$
- $D (-syt, 0)$
- $A (syt, 0)$
- $-\sigma_1$
- $-syt - \tau$
- $45°$ τ
- $syt + \sigma_1$
- $-\tau$
- $B (0, -syt)$
- $-syt$
- $\sigma_2 = -syt$
- Tresca's Hexagone
- $-\sigma_2$
- $\sigma_1 = -\sigma_2$
- $\sigma_2 = -\sigma_1$
- Shear diagonal
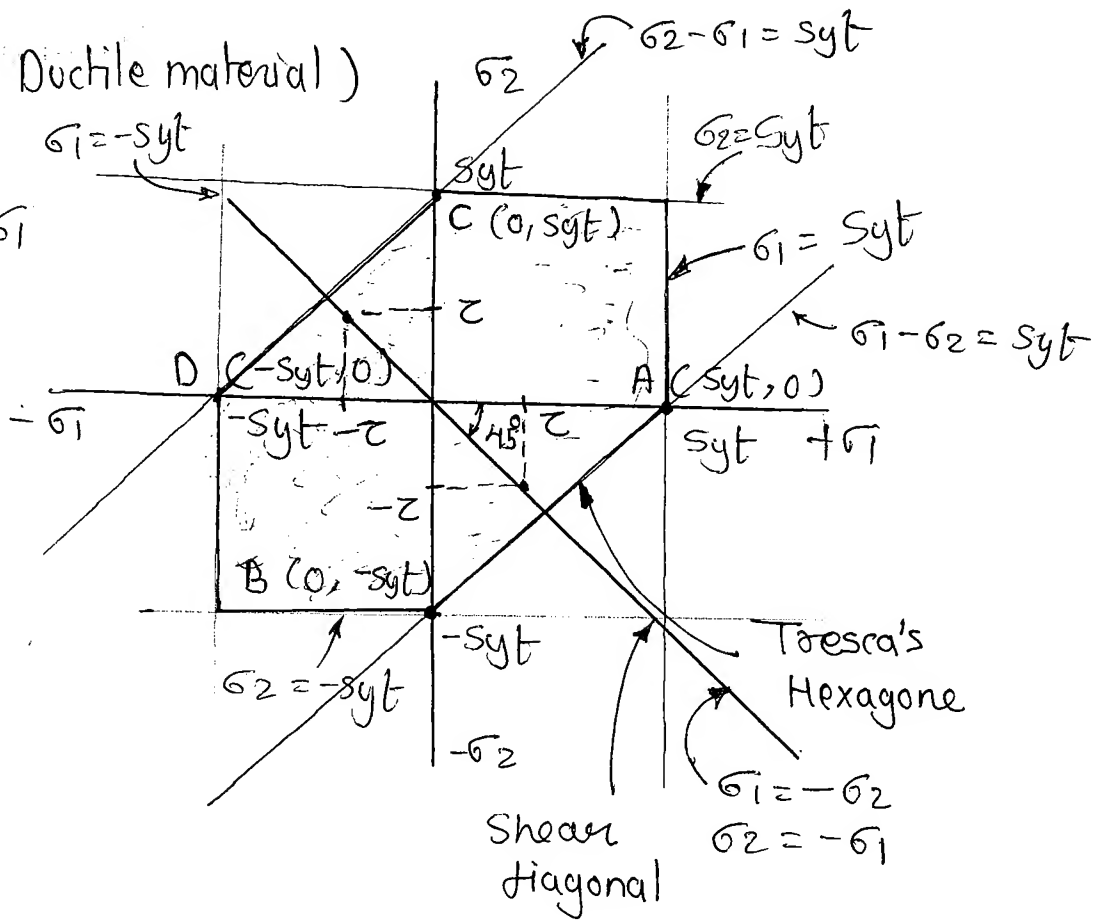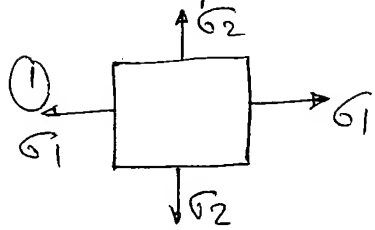
a] If $\sigma_1 > \sigma_2$

then, $\tau_{max} = \dfrac{\sigma_1}{2}$

$\tau_{max} = \dfrac{\sigma_1}{2} = \dfrac{syt}{2}$

$\sigma_1 = syt$

b] If $\sigma_2 > \sigma_1$

$\tau_{max} = \dfrac{\sigma_2}{2} = \dfrac{syt}{2}$

$\sigma_2 = syt$

② element diagram (σ_2 down, σ_1 in, σ_1 in, σ_2 up)

c] If $\sigma_1 > \sigma_2$

$\tau_{max} = \dfrac{\sigma_1}{2} = \dfrac{-syt}{2}$

$\sigma_1 = -syt$

d] If $\sigma_2 > \sigma_1$

$\tau_{max} = \dfrac{\sigma_2}{2} = \dfrac{-syt}{2}$

③ element diagram (σ_2 down, σ_1 right, σ_1 left, σ_2 up)
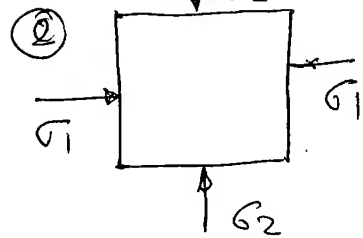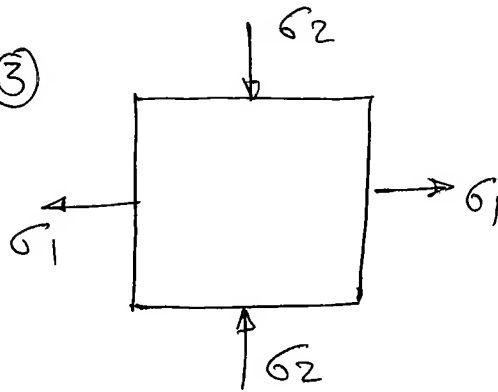
$\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{syt}{2}$

$\sigma_1 - \sigma_2 = syt$

If $\sigma_2 = 0$, $A (syt, 0)$

$\sigma_1 = 0$, $B (0, -syt)$

$$\tau_{max} = max\ of \begin{cases} \dfrac{\sigma_1 - \sigma_2}{2} & 40 \\[2mm] \dfrac{\sigma_1 - \sigma_1}{2} & 50 \\[2mm] \dfrac{\sigma_2 - \sigma_3}{2} & 10 \end{cases}$$

20   100

④
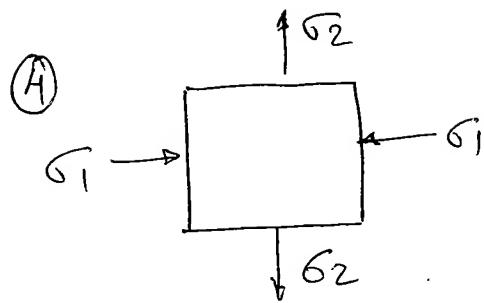
$$Z_{max} = \frac{\sigma_2 - \sigma_1}{2} = \frac{S_{yt}}{2}.$$

$$\sigma_2 - \sigma_1 = S_{yt}$$

If,

$\sigma_1 = 0$. $C(0, S_{yt})$

$\sigma_2 = 0$ $D(-S_{yt}, 0)$.

* State of pure shear

$$\sigma = \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sigma_1}{2} & \tau_{xy} \\ \tau_{xy} & -\frac{\sigma_2}{2} \end{bmatrix}$$

$$\sigma = \begin{bmatrix} 0 & \tau \\ -\tau & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sigma_1}{2} & 0 \\ 0 & \frac{\sigma_2}{2} \end{bmatrix}$$

A component subjected to complex state of stress is said to be failed when the maximum shear stress at a point in the material exceeds the shear strength at yield point under simple tension.

Q A component is loaded as shown in fig the factor of safety according to

1] Maximum normal stress theory.

a) 1   b) 2   c) 3   d) 4.

100 mm²

100 MPa

$Syt = 300\ MPa$.

$$Syt = \frac{\sigma}{F.S}$$
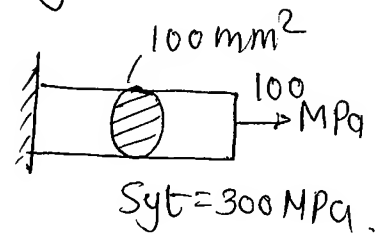
2] Maximum shear stress theory

a) 1   b) 2   c) 3   d) 4

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{Syt}{2 \times FS}$$

Q A steel bolt is subjected to a direct tensile load of 8.4 kN and a shear load of 48 kN. The cross section area of bolt is 600 mm² and Syt for bolt material is 200 MPa, the F.S according to maximum shear stress theory is

a] 1   b] 2   c] 3   d] 4.

$$\sigma_1 = \frac{8.4 \times 10^3}{600} = 14\ 80 MPa$$

$$\tau = \sigma_S = \frac{18 \times 10^3}{600} = 30\ MPa.$$
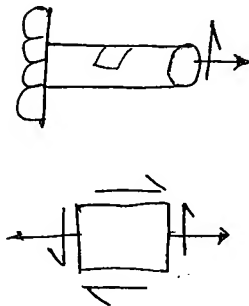
$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}.$$

$$= 90\ MPa.$$

$$\sigma_2 = -10\ MPa.$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{90 - (-10)}{2} = 50\ MPa.$$

$$\tau_{max} = \frac{Syt}{2FS}$$

$$50 = \frac{200}{2FS} \quad \boxed{FS = 2}$$

③ Saint Venant's theory (or) Maximum Principal strain theory.

If $\sigma_1 > \sigma_2 > \sigma_3$

Then $\varepsilon_1 > \varepsilon_2 > \varepsilon_3$.

$\varepsilon_1 \rightarrow$ Maximum principal strain

$\varepsilon_1 > \dfrac{S_{yt}}{E}$ — Failed.

$\mu = \dfrac{-\text{lateral strain}}{\text{longitudinal } ,,}$

$\mu = 0$ for cork

$\mu = 0.25$ to 0.33 for steel.

$= 0.47$ for rubber

Design Equation

$$\varepsilon_1 = \dfrac{S_{yt}}{E \times F_s} \qquad - ①$$

$$\varepsilon_1 = \dfrac{\sigma_1}{E} - \dfrac{\mu\sigma_2}{E} - \dfrac{\mu\sigma_3}{E} \qquad - ②$$

$$\dfrac{\sigma_1}{E} - \dfrac{\mu\sigma_2}{E} - \dfrac{\mu\sigma_3}{E} = \dfrac{S_{yt}}{E \times F \cdot S}$$

$$\boxed{\sigma_1 - \mu\sigma_2 - \mu\sigma_3 = \dfrac{S_{yt}}{F \cdot S}}$$

For 2D $\sigma_3 = 0$

$$\sigma_1 - \mu\sigma_2 = \dfrac{S_{yt}}{F \cdot S}$$

Not valid for Brittle material or for Ductile material.
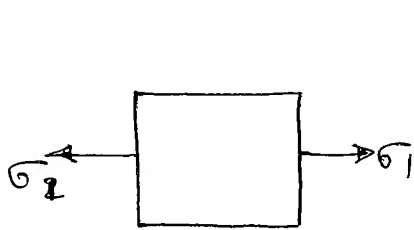
$$\varepsilon_{uni} = \frac{\sigma_1}{E}$$
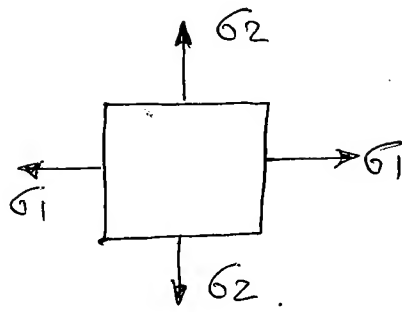
$$\varepsilon_{biaxial} = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E}$$

$$\varepsilon_{uni} > \varepsilon_{bi}$$

Graph
(Ductile)

a] If $\sigma_1 > \sigma_2$

$$\varepsilon = \frac{\sigma_1 - \mu \sigma_2}{E} = \frac{syt}{E}$$

$$\sigma_1 - \mu \sigma_2 = syt$$

If $\sigma_1 = 0$ $A\left(0, -\frac{syt}{\mu}\right)$

     $\sigma_2 = 0$ $B(syt, 0)$

b] If $\sigma_2 > \sigma_1$.

$$\sigma_2 - \mu \sigma_1 = syt$$

If $\sigma_1 = 0$ $c(0, syt)$

     $\sigma_2 = 0$ $D\left(-\frac{syt}{\mu}, 0\right)$

$\left(\frac{syt}{1-\mu}, \frac{syt}{1-\mu}\right)$

$+\sigma_2$

$E\left(0, \frac{syt}{\mu}\right)$

$\left(\frac{-syt}{1+\mu}, \frac{syt}{1+\mu}\right)$ $C(0, syt)$

$F\left(-\frac{syt}{\mu}, 0\right)$ $B(syt, 0)$

$D\left(-\frac{syt}{\mu}, 0\right)$ $+\sigma_1$

$H\left(\frac{syt}{\mu}, 0\right)$

$G(0, -syt)$

$\left(\frac{syt}{1+\mu}, \frac{-syt}{1+\mu}\right)$

$\left(\frac{-syt}{1-\mu}, \frac{-syt}{1-\mu}\right)$

$A\left(0, -\frac{syt}{\mu}\right)$

$-\sigma_2$

②

c] If $\sigma_1 > \sigma_2$.

$$\varepsilon_1 = \frac{\sigma_1}{G} - \frac{\mu\sigma_2}{E} = \frac{Syc}{E}$$

$$\sigma_1 - \mu\sigma_2 = -Syt$$

If $\sigma_1 = 0 \ E\left(0, +\frac{Syt}{\mu}\right)$

$\sigma_2 = 0 \ F(-Syt, 0)$

D] If $\sigma_2 > \sigma_1$

$$\sigma_2 - \mu\sigma_1 = -Syt$$

If $\sigma_1 = 0 \ G(0, -Syt)$
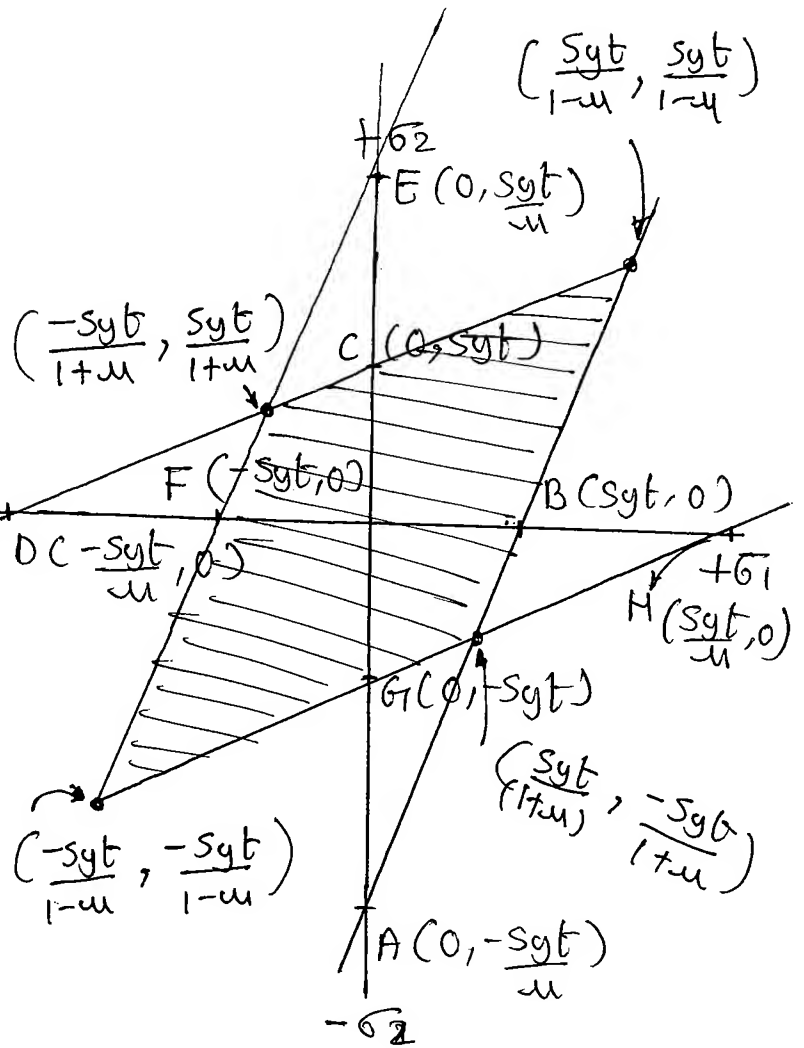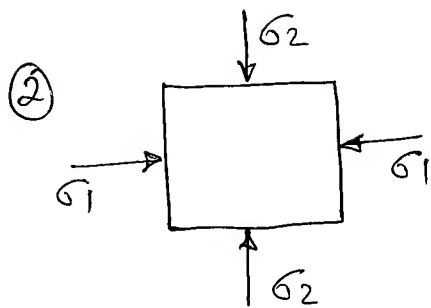
$\sigma_2 = 0 \ H\left(\frac{Syt}{\mu}, 0\right)$

A component subjected to complex state of stress is said to be failed when the maximum principal strain at a point in the material exceeds the strain at yield point under simple tension.

(A) Haigh's theory (or) Maximum strain energy theory.

Strain Energy

$$U = \frac{1}{2} \varepsilon \times \sigma$$

$$= \frac{1}{2} \sigma \varepsilon$$

$$= \frac{1}{2} \times \sigma \times \frac{\sigma}{E}$$

$$= \frac{\sigma^2}{2E}$$

stress.
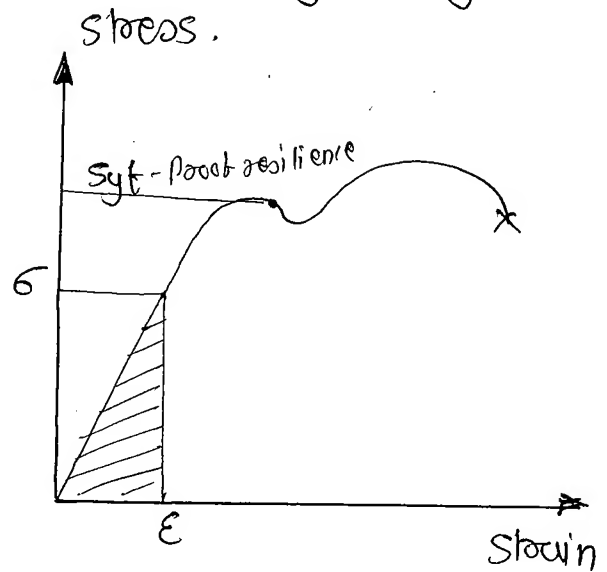
$$U \text{ at yield point} = \frac{S_{yt}^2}{2E}$$

Strain energy density under complex loading.
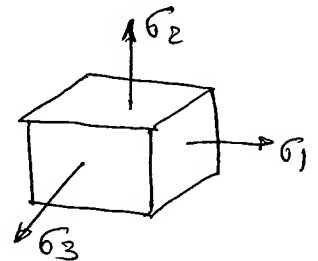
$$U_{complex} = \frac{1}{2}\sigma_1 \varepsilon_1 + \frac{1}{2}\sigma_2 \varepsilon_2 + \frac{1}{2}\sigma_3 \varepsilon_3 .$$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E} - \frac{\mu \sigma_3}{E}$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\mu \sigma_2}{E} - \frac{\mu \sigma_1}{E}$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \frac{\mu \sigma_1}{E} - \frac{\mu \sigma_2}{E} .$$

$$U_{complex} = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1) \right]$$

$$U_{complex} \leq U \text{ simple tension} \quad - \quad \text{safe.}$$

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1) \right] = \left(\frac{S_{yt}}{F \cdot S}\right)^2 \times \frac{1}{2E}.$$

$$\frac{S_{yt}}{F \cdot S} = \sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)}$$

For 2D

$$\frac{Syt}{F\cdot S} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2}$$

$$x^2 + y^2 - 2bxy = a^2.$$

$$\sigma_1^2 + \sigma_2^2 - 2\mu_1\sigma_1\sigma_2 = Syt^2.$$

If $\sigma_1 = 0$ $\sigma_2 = \pm Syt$

   $\sigma_2 = 0$ $\sigma_1 = \pm Syt$

at $\mu = 0.3$
0.82 Syt
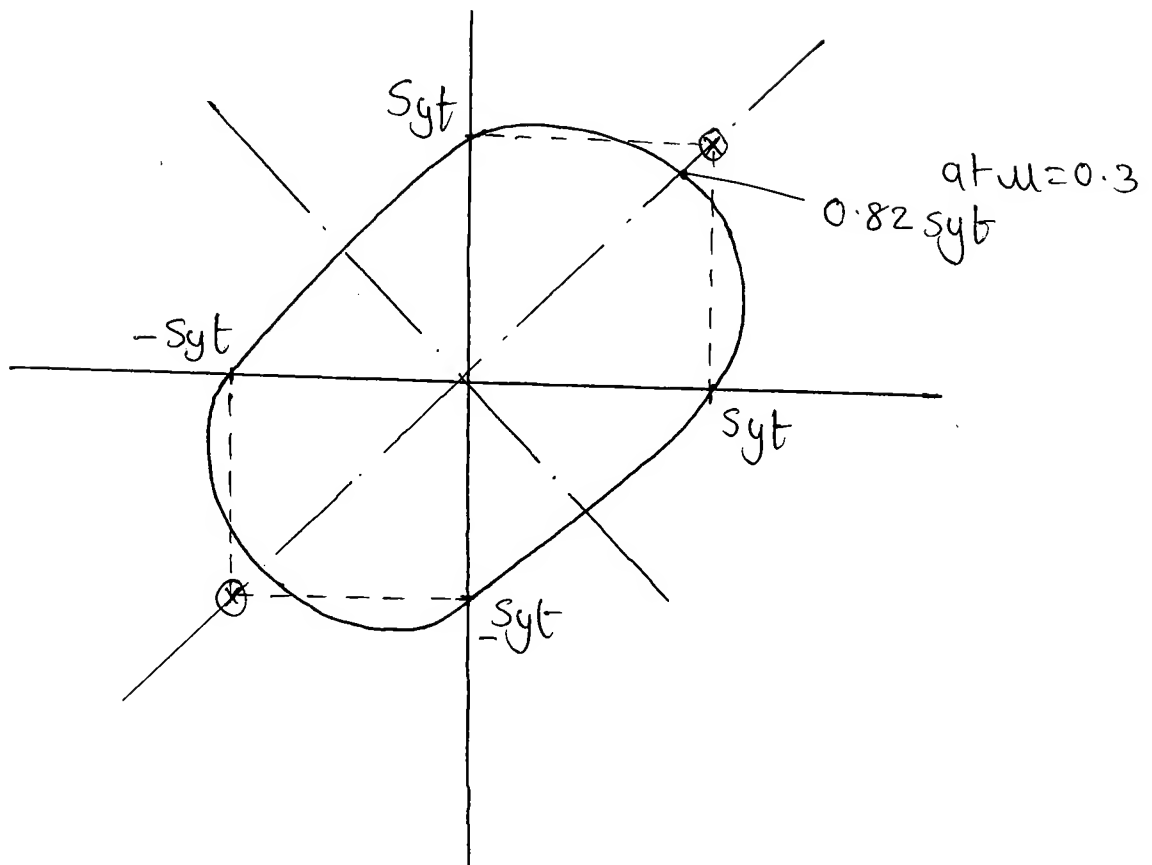
Validity — Valid for Ductile material.

Q If the principal stresses at point are $\sigma_1 = +2f$
$\sigma_2 = +f$, $\sigma_3 = -\frac{1}{2}f$, and $S_{yt} = 200 \, N/mm^2$, $\mu = 0.3$
The value of 'F' according to Haish's theorem

a) $100 \, N/mm^2$ b] $75 \, N/mm^2$ c] $90 \, N/mm^2$ ✓ d] $150 \, N/mm^2$

$$\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\mu \left(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1 \right) \right]^{1/2} = \frac{S_{yt}}{f \cdot S}$$

5] Von Mises and Henky's theory (or) Shear strain Energy theory (or) Distortion Energy theory.
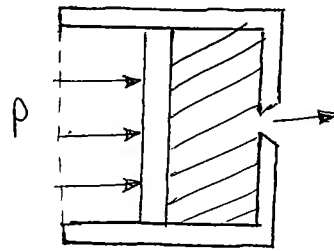
Total Strain Energy (Ut)

$$U_t = \frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu \left(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1 \right) \right]$$

$$U_t = U_v + U_d$$
= Volume change + Change of shape.

$$= \text{Volume change} + \text{Distortion} + \text{Distortion}$$

$$\sigma = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \overset{\text{shear}}{}$$

$$E = 3K(1-2\mu)$$

$$= 3 \frac{\sigma}{\varepsilon_V}(1-2\mu)$$

$$= \begin{bmatrix} \sigma/3 & 0 \\ 0 & -\sigma/3 \end{bmatrix}$$

$$\varepsilon_V = 3\frac{\sigma}{E}(1-2\mu) \quad \cdots \quad \varepsilon_V = \frac{\delta V}{V}$$
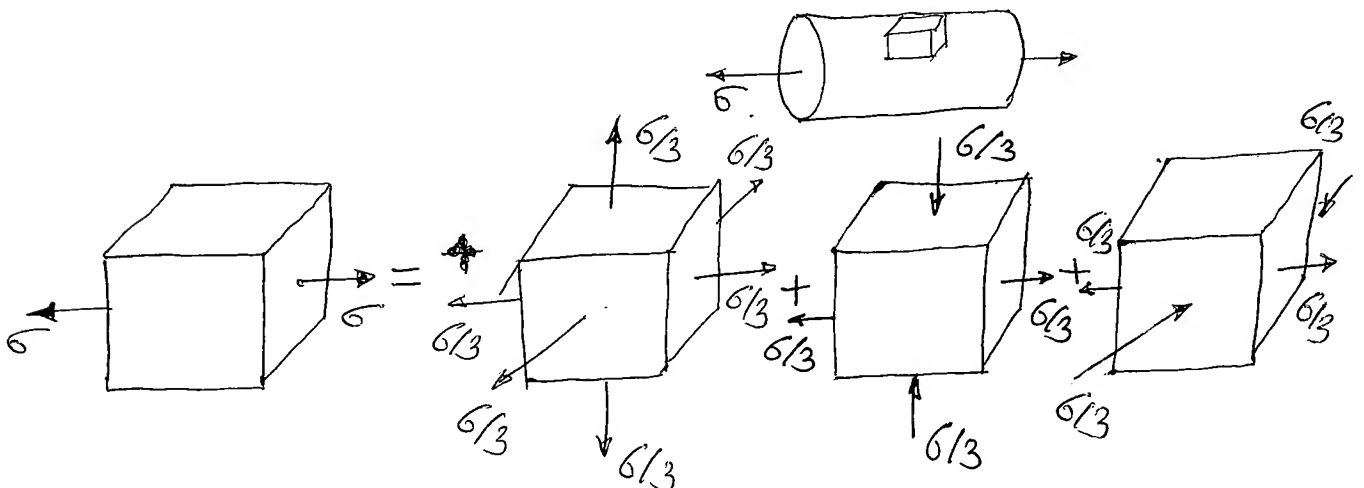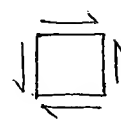
$$\delta V = \frac{3\sigma}{E}(1-2\mu)V$$

If $\mu = 0.5$, $\delta V = 0$.

$$U_t = \frac{1}{2E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

If $\mu = 0.5$ $\quad U_t = U_d$.

$$U_d = \frac{1}{2E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$= \frac{(1+\mu)}{3E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$E = 2G(1+\mu)$$

$$= \frac{1}{6G}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

For simple tension.

$$\sigma_1 = \sigma \quad , \quad \sigma_2 = \sigma_3 = 0.$$

$$U_d = \frac{1}{6G}\sigma^2.$$

$$E = 2G(1+\mu)$$

$$U_d = \frac{1+\mu}{3E}\sigma^2.$$

Distortion energy under Complex loading $>$ Distortion energy under simple tension at yield point

— Failed

$$= \frac{1}{6G} \, S_{yt}^2 \quad \ldots \ldots \text{At yeiled point}$$

$$\frac{1}{6G} \left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right] = \frac{1}{6G} \left(\frac{S_{yt}}{F \cdot S} \right)^2$$

$$\frac{S_{yt}}{F \cdot S} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

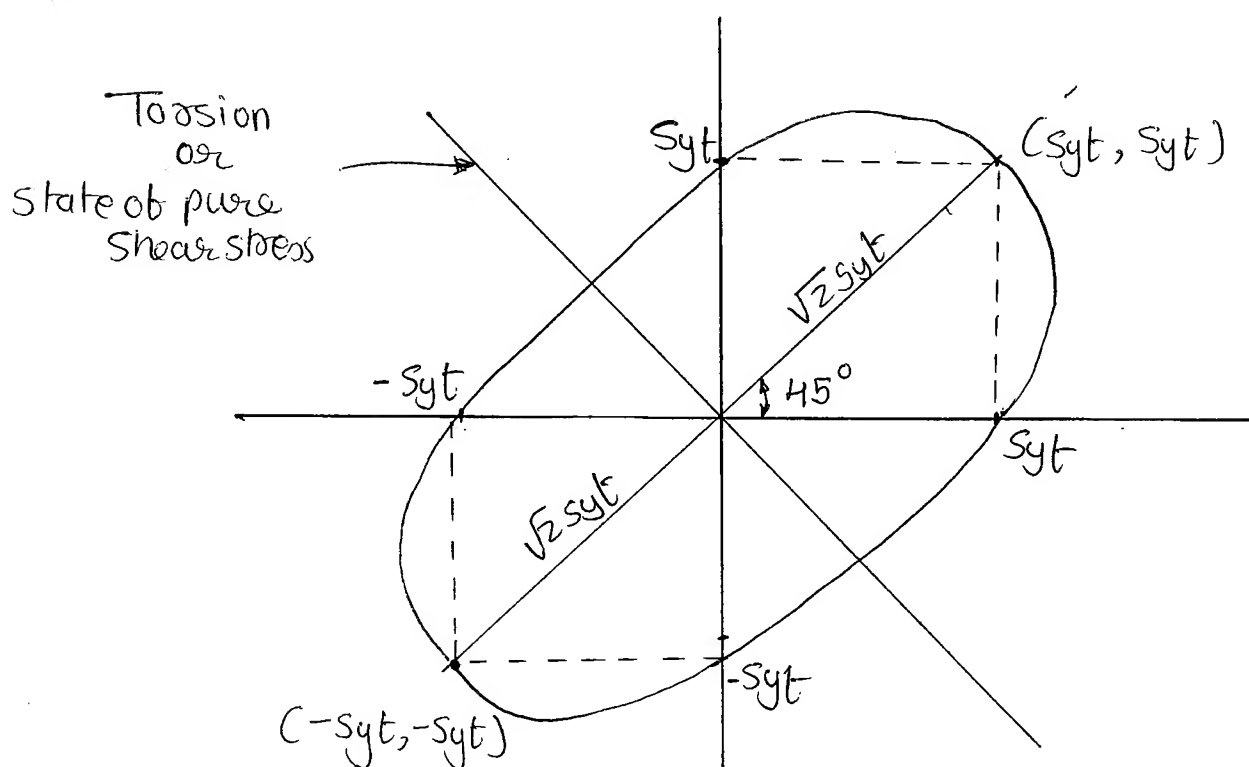For 2D stress.

$$\frac{S_{yt}}{F \cdot S} = \sqrt{(\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)}$$

$$x^2 + y^2 - xy = a^2 \quad \ldots \text{Ellipse eq}^n.$$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = S_{yt}^2$$

$$\sigma_1 = 0 \qquad \sigma_2 = \pm S_{yt}$$
$$\sigma_2 = 0 \qquad \sigma_1 = \pm S_{yt}$$

Torsion or State of pure Shear stress

Validity — This theory gives very good results for

① $\sigma_1 = \dfrac{Syt}{F \cdot S}$ — Rankine

② $\dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{Syt}{F \cdot S}$ — Guest

③ $\sigma_1 - \mu\sigma_2 = \dfrac{Syt}{F \cdot S}$ — St venent

④ $\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = \left(\dfrac{Syt}{F \cdot S}\right)^2$ — Haigh's

⑤ $\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \left(\dfrac{Syt}{F \cdot S}\right)^2$ — Von Moises

Q   A component is subjected to torsion, T. Estimate the permisseble value of shear stress z according to different theories of Failure in terms of yield strength, Syt. Take $\mu = 0.3$ E $= 2.1 \times 10^5$ N/mm²

Ans
① Rankines $\sigma_1 = \dfrac{Syt}{F \cdot S}$ =

$$\sigma = \begin{bmatrix} 0 & z \\ z & 0 \end{bmatrix} = \begin{bmatrix} z & 0 \\ 0 & -z \end{bmatrix}$$

$\sigma_1 = Syt$
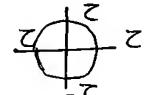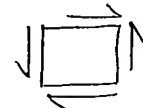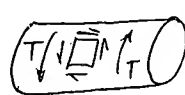
$z = Syt$

$\dfrac{z}{Syt} = 1$

Exp. value → $Ssy = 0.57$ to 0.6 of Syt

② Guest   (most conservative theory)

$\dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{Syt}{2 F \cdot S}$

$z - (-z) = Syt$

$2z = Syt$ $z = \dfrac{Syt}{2} = 0.5 Syt$

③ St venents theory

$$\sigma_1 - \mu\sigma_2 = S_{yt}$$
$$2 - 0.3 \times (-2) = S_{yt}$$
$$2 = \frac{S_{yt}}{1.3}$$
$$2 = 0.769 \, S_{yt}, \quad \frac{2}{S_{yt}} = 0.769$$

④ Haigh's theory

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = S_{yt}^2$$
$$2 + (-2)^2 - 2\mu(2)(-2) = S_{yt}^2$$
$$2 = 0.63 \, S_{yt}$$

⑤ Von moises.

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = S_{yt}^2$$
$$2^2 + (-2)^2 - 2(-2) = S_{yt}^2$$
$$2 \cdot 2^2 + 2^2 = S_{yt}^2 \qquad (\text{more economical})$$
$$2 = 0.577 \, S_{yt}$$

Note:–
1. Use of guest theory increase size. hence increase more cost and create bulky parts

Bending Equation.

$$M = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$

M_x → Torsion.

M_y, M_z → Bending.

$$M = M_x i + M_z k$$
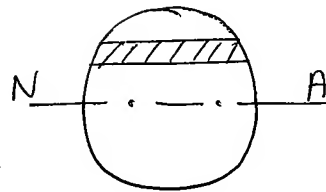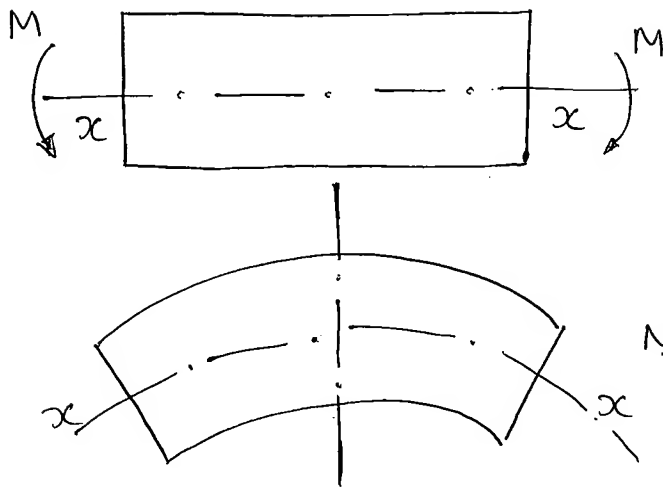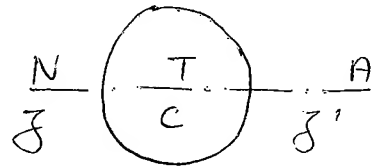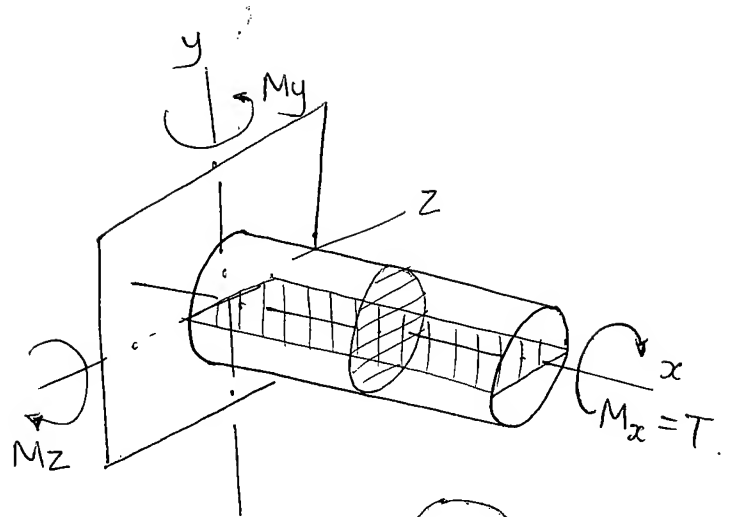
$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}.$$

M = Bending moment

I = M.I of c/s about neutral axis

σ_b = Bending stress

y = Distance measured from Neutral axis.

R = Radius measured from neutral axis.

$$I = \frac{\pi}{64} d^4$$

$$= \frac{\pi}{64} (D_o^4 - d_i^4)$$

Max stress σ_{b} max

min stress $=$ zero $\qquad$ d $\sigma_{b}=0$ $\sigma_{b} \propto y$

σ_{b} max

If $y=r$ $\sigma_{b}=\dfrac{r}{R}\sigma$ max

$y=R$ & $\sigma_{b}=\sigma_{b}$ max.

N · A.
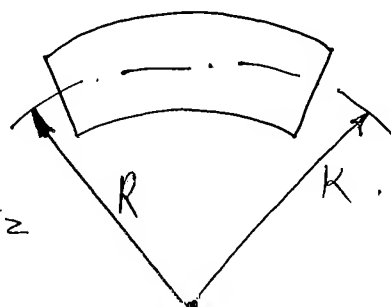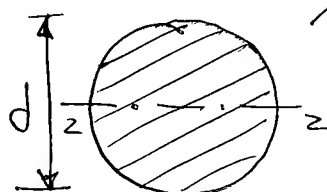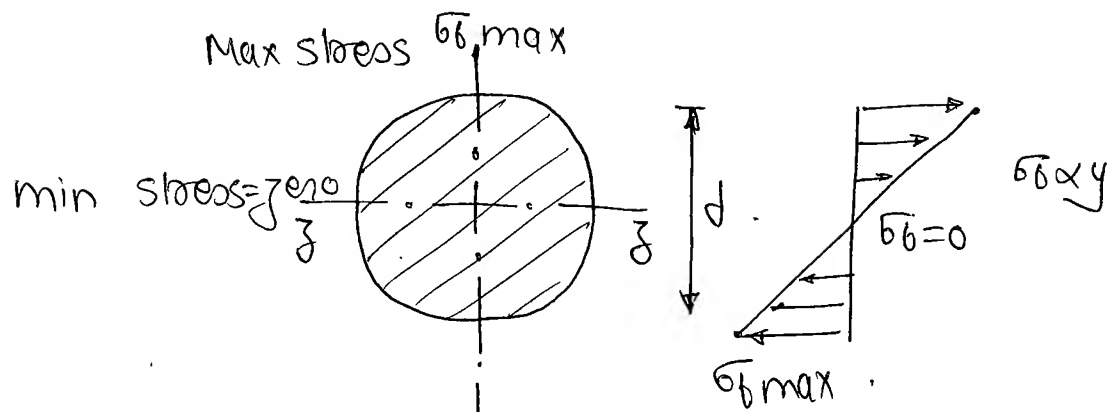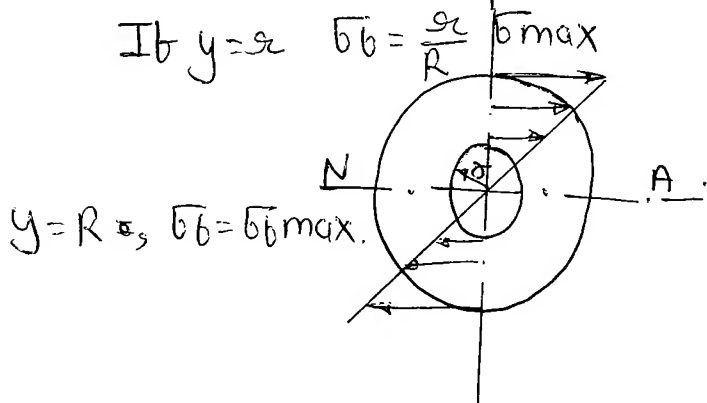
Along the N-A. Hence σ_{b} min is zero.

Torsion Equation.

$Z \propto r$ $$\boxed{\dfrac{T}{J}=\dfrac{Z}{r}}=\dfrac{G\theta}{\ell}$$
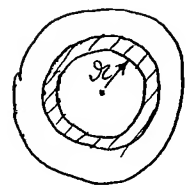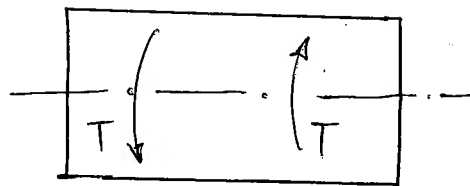
T = Torsion

J = Polar moment of Inertia.

$\quad = I_{xx}+I_{yy} = \dfrac{\pi}{32}d^4$

$\quad = \dfrac{\pi}{32}(D^4-d^4)$

Z = Shear stress at radius R

G = Modulus of rigidity

θ = Angle of twist.

ℓ = length of member under torsion

$x=R$ τ_{max} $\tau = \tau_{max}$ $x=\gamma_{max}$

$x_0, \tau=0$ R τ_{min}

$$\tau_{min} = \frac{x}{R}\,\tau_{max}$$
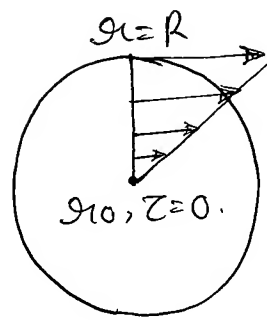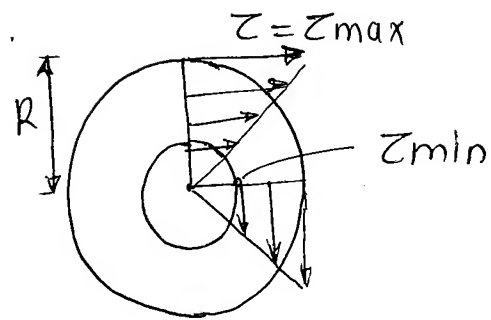
For solid.   For hollow.

Von mises eq^n for Axial and torsion loading.
(Bending)

A σ_b B σ_b M A T B T M
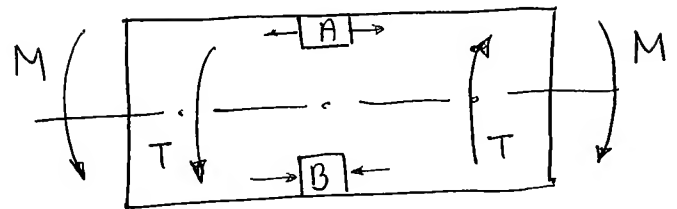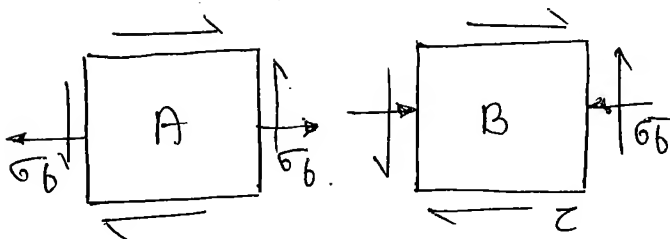
$$\left[\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2\right]^{1/2} = \frac{S_{yt}}{F\cdot S} = \sigma \text{ equivalent.}$$

for A.

$$\sigma x = \sigma$$
$$\sigma y = 0.$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}.$$

$$\sigma_1 = \frac{\sigma}{2} + R$$

$$\sigma_2 = \frac{\sigma}{2} - R.$$

        put in eq^n

$$\left[\left(\frac{\sigma}{2}+R\right)^2 + \left(\frac{\sigma}{2}-R\right)^2 - \left(\frac{\sigma}{2}+R\right)\left(\frac{\sigma}{2}-R\right)\right]^{1/2} = \sigma \text{ equivalent.}$$

$$\left[\left(\frac{\sigma}{2}\right)^2 + 3R^2\right]^{1/2} = \sigma \text{ eq}$$

$$\left[\left(\tfrac{\sigma}{2}\right)^2 + 3\left\{ \sqrt{\left(\tfrac{\sigma}{2}\right)^2 + z^2} \right\}^2 \right]^{1/2} = \sigma_{eq}$$

$$\left[\left(\tfrac{\sigma}{2}\right)^2 + 3\left(\tfrac{\sigma}{2}\right)^2 + 3z^2 \right]^{1/2} = \sigma_{eq} \, .$$

$$\left[4\left(\tfrac{\sigma}{2}\right)^2 + 3z^2 \right]^{1/2} = \sigma_{eq} \, .$$

$$2\tfrac{\sigma}{2} + z\,(1+3)^{1/2} = \sigma_{eq} \, .$$

$$\boxed{\sigma_{eq} = \sqrt{\sigma^2 + 3z^2} \, .}$$

Q: The homogeneous state of stress for a metal part undergoing plastic deformation is

$$T = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ 10 & 5 & 0 \\ \tau_{xy} & \sigma_{yy} & \sigma_{yz} \\ 5 & 20 & 0 \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \\ 0 & \sigma_{zy}\,0 & -10 \end{bmatrix}$$

where the stress component value are in MPa. using Von moises yield criterism.

a] 9·50   b] 16·07   c] 28·52   d] 19·41

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \, .$$

$$= \frac{10+20}{2} + \sqrt{\left(\frac{10-20}{2}\right)^2 + 5^2}$$

$$= 22 \cdot 071 \, .$$

$$\sigma_2 = \frac{\sigma_y + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + \tau_{yz}^2}$$

$$= \frac{20 - 10}{2} + \sqrt{15^2 + 5^2}$$

$$= 7 \cdot 93 \, .$$

$$\sigma_3 = -10\,\text{mpa}.$$

$$\frac{S_{yt}}{FS} = \sigma_{eq} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_2 + \sigma_3\sigma_1)}$$

$$= 27.83.$$

$$Z = 0.557\,\sigma_{eq}.$$

$$= 0.557 \times 27.83$$

$$= 16.057.$$

⑥. Mohr's Theory (Coloumb - Mohr's theory)
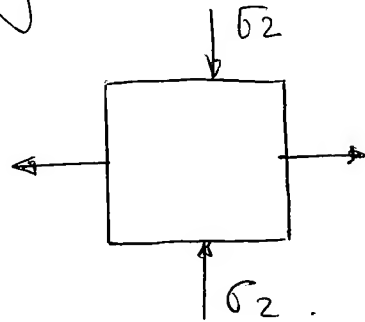
For Duchle $S_{yt} = S_{yc}$ (isotropic - Assumption)

$$\sigma_1 - \sigma_2 = \frac{S_{yt}}{FS}.$$

$$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_2}{S_{yt}} = \frac{1}{FS}$$

For Brittle Material ($S_{yt} \neq S_{yc}$)

$$\frac{\sigma_1}{S_{yt}} - \frac{\sigma_2}{S_{yc}} = \frac{1}{F.S}.$$
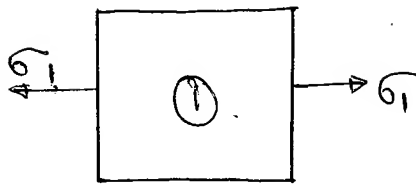
$$\frac{\sigma_{1\,su}}{S_{ut}} - \frac{\sigma_2}{S_{ut}} = \frac{1}{F \cdot S}$$

$$\sigma_2 = 0.$$
$$\sigma_1 = S_{ut}.$$

$$\frac{-\sigma_1}{S_{ut}} - \frac{\sigma_2}{S_{uc}} = \frac{1}{F \cdot S}$$

$$\sigma_2 = 0.$$
$$S_{ut} = -\sigma_1$$

Pure shear

Failure envelope

Safe Zone

S_{uc}

$S_{uc} > S_{ut}.$

Design for Brittle material

Q A steel pin subjected to a shear force 25kN and the direct compression of 41 kN. The permissible compressive stresses in pin material is 50MPa, then the suitable dia of pin according to distortion energy theory is

a] 75.5 mm   b] 38 mm   c] 20mm   d] 45mm .

$$T = \frac{\pi}{16} f_s d^3$$

$f_s = 25 KN$.

$F_t = 41 KN$.

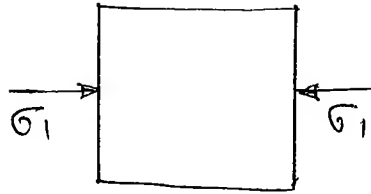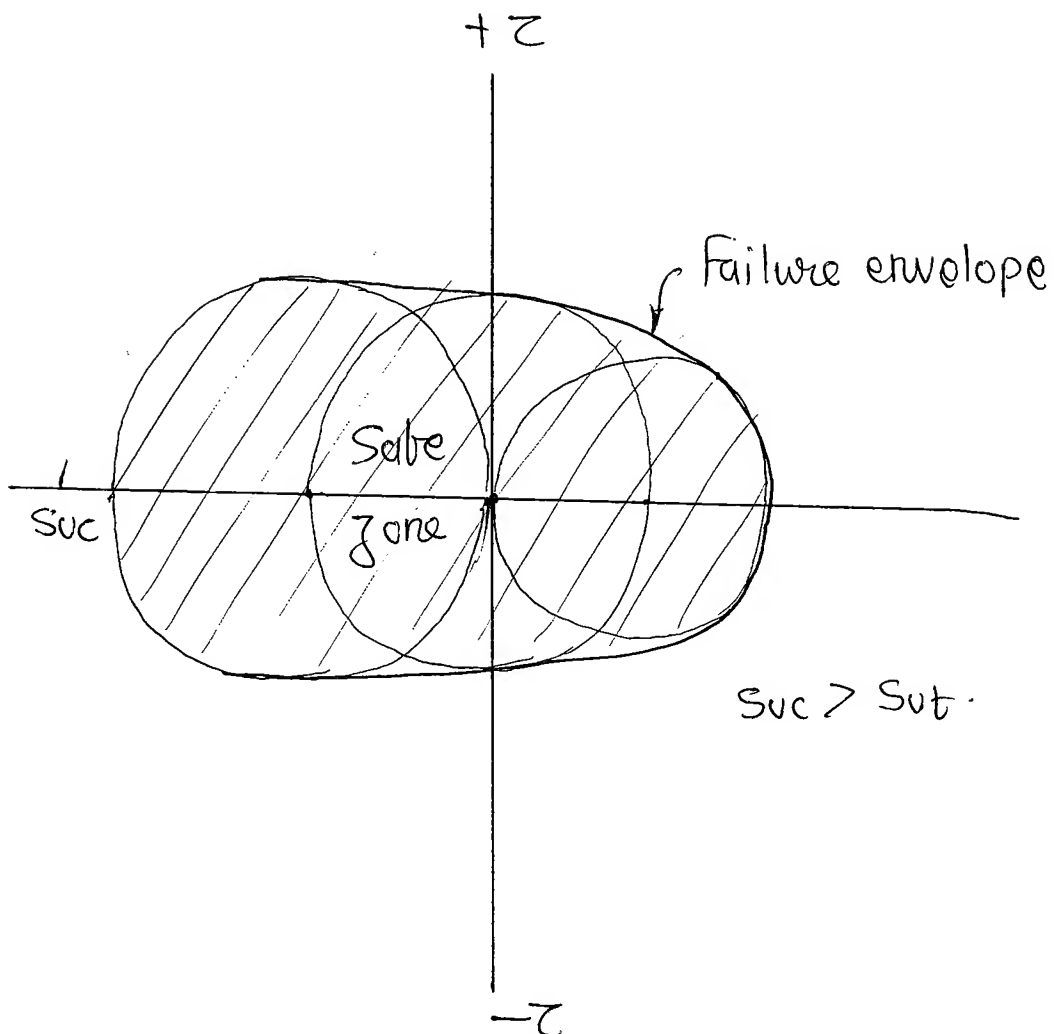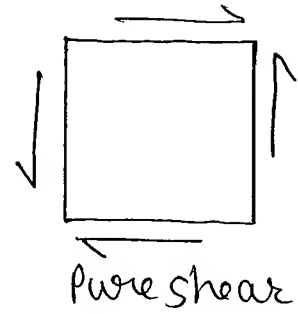$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2}$.

$$50 = \sqrt{\left(\frac{41}{A}\right)^2 + 3\left(\frac{25}{A}\right)^2} = 59.63$$

$$= \sqrt{\frac{41^2 + 3\times 25^2}{A^2}}$$

$$A = \frac{2\sqrt{889}}{50}$$

$$A = 1192.64$$

$$\frac{\pi}{4} d^2 = 1192.64$$

$$\boxed{d = 38 \, mm}$$

Q A load P 45 KN is applied to a crank shaft of dia. 90mm at a distance of 200mm from bearing. The material is 30 C4 with $S_{yt} = 315$ MPa. Factor of safety according to Guest's theory is

a] 1   b] 2   c] 3   d] 4 .

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{S_{yt}}{2 FS}$$

(torsion) $T = P \times 150$
$= 45 \times 150$
$= 6750 \, KN\text{-}mm$

(Bending) $M = P \times 200$
$= 45 \times 200$
$= 9000 \, KN\text{-}mm$

$T = \frac{\pi}{16} f_s d^3$

load p is applied ⊥ or to plane of paper.

150

200

P
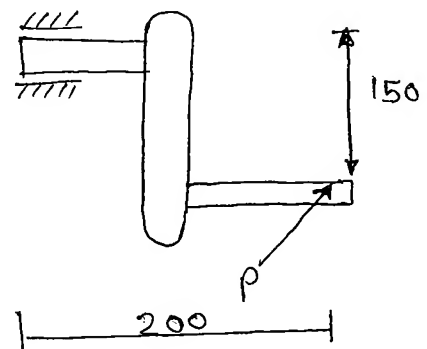
0.047
$4.044 \, KN\text{-}mm$

$$\sigma_b = \frac{32 M}{\pi d^3}$$

$$= \frac{32 \times 9000}{\pi \times 90^3}$$

$$= 0.125 \ KN/mm^2$$

$$= 125.75 \ N/mm^2$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{125.75}{2}\right)^2 + (47.15)^2}$$

$$= 78.59$$

$$\tau_{max} = \frac{S_{yt}}{2 \times F \cdot S}$$

$$78.59 \times 2 \times FS = 315$$

$$\boxed{FS = 2}$$

S A component subjected to Fluxtuating stresses

Desing for Fatigue.

A component is subjected to a fluctuating stresses fails at a stress much below their static strength. This phenomenon is called as Fatigue.

wohler - Fatigue.

Residual energy.
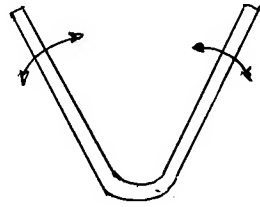
Ex:- 100 KN - strength.
10 KN - Residual ↑ then strength become 90 KN ↓

Terminology.

i]

σ_a

σ_{max} σ_{min} σ_m
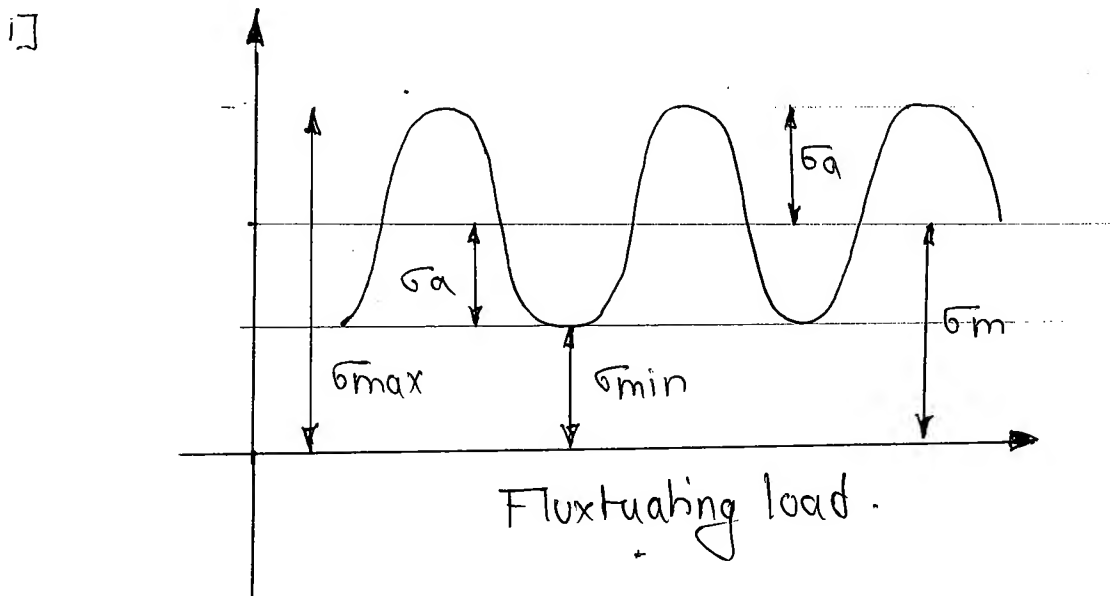
Fluxtuating load.

mean stress, $\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$

Stress amplitude $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$

Stress Ratio, $R = \dfrac{\sigma_{min}}{\sigma_{max}}$.

2]

Repeated stress $(\sigma_m = \sigma_a)$

$\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2} = \dfrac{\sigma + 0}{2} = \sigma/2$.

$\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{\sigma + 0}{2} = \sigma/2$

$R = \dfrac{\sigma_{min}}{\sigma_{max}} = 0$.
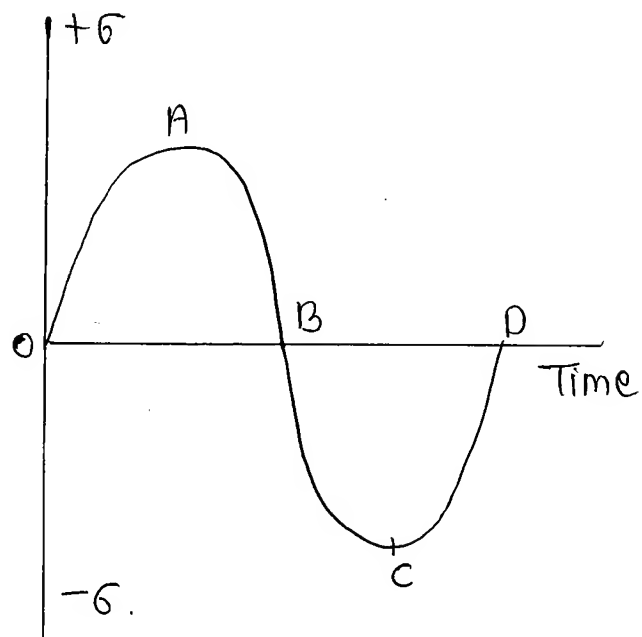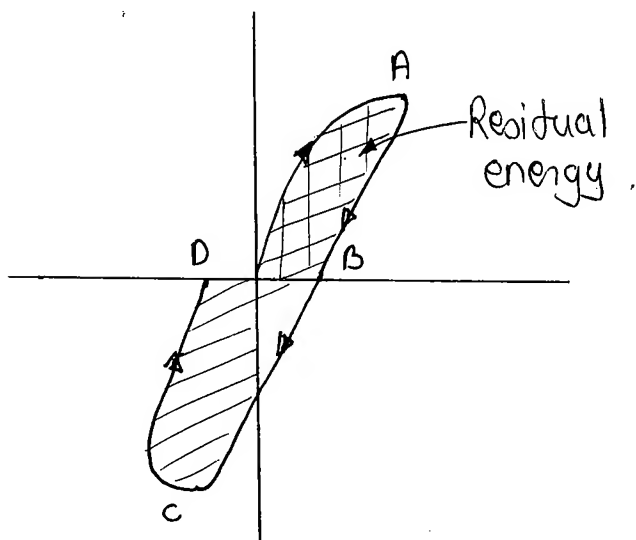
3]

Completely Reversible

$\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2} = \dfrac{\sigma + (-\sigma)}{2} = 0$

$\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{\sigma - (-\sigma)}{2} = \dfrac{2\sigma}{2} = \sigma$.

$R = \dfrac{\sigma_{min}}{\sigma_{max}} = \dfrac{-\sigma}{\sigma} = -1$.

a] Rail Road car axle.

b] IC Engine

Pressure in IC Engine

c]

Gears

D]

h

Fluctuating flag.

E.

wheel.
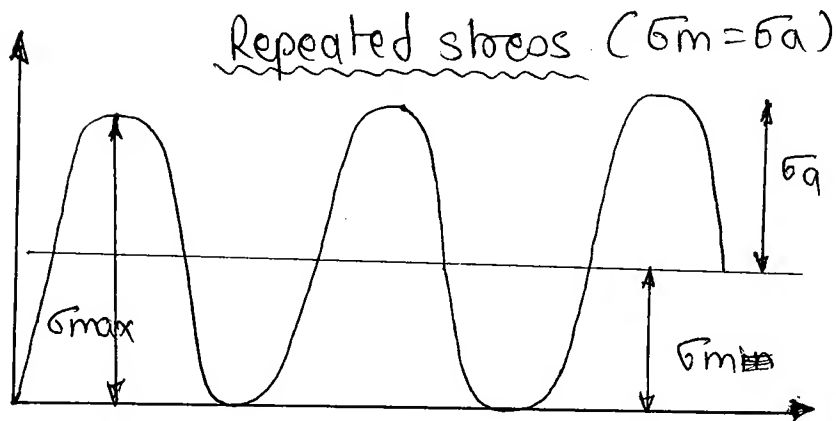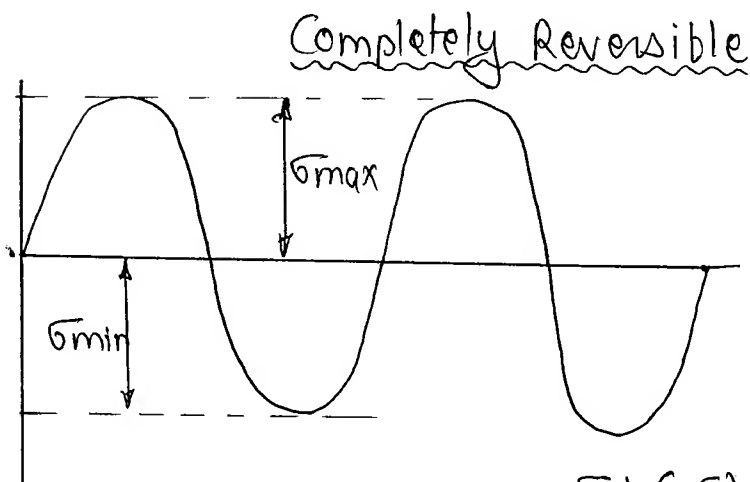
Axle → stationary or rotating member and is loaded in bending.

a]

$W/2$ $W/2$
a   a

m max.

$\frac{W}{2} \times a$ $\frac{W \times a}{2}$ B·M

Pure Bending $W/2$

$W/2$ S·F

$$\sigma_b = \frac{32 \, M_{ax}}{\pi d^3}$$

comp.

$+\sigma_b$

B] **I C Engine.**

Pr.

cycle continue hence Repeated load.

85mm

$i = 1, 2, \ldots$ (variable)

1725 rpm

Fi

7·6mm

R = 250mm

Motor

revolution counter.

Fi

Mmax.

$$\sigma_b = \frac{32M}{\pi d^3} = \sigma_a.$$

σ_a

$\sigma_m = 0$

load		speed rpm
σ_{a1} =	F_1	N_1
σ_{a2} =	F_2	N_2
σ_{a3} =	F_3	N_3
:		:

(S-N curve)

Stress amplitude
↑ Static load

σ_b. Sut

log (Stress)

low cycle Fatigue

High cycle Fatigue →

Sf

Se'

Finite life ←→ Infinite life.

$10^0 \quad 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8$ → No of revolution

log (speed) N.
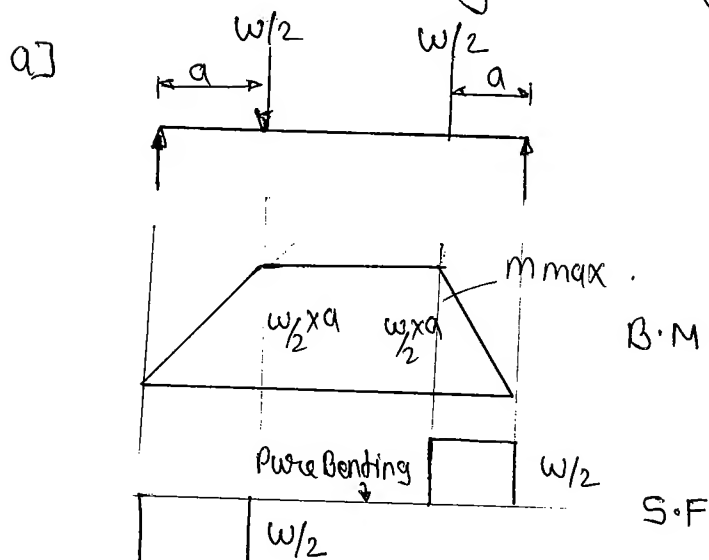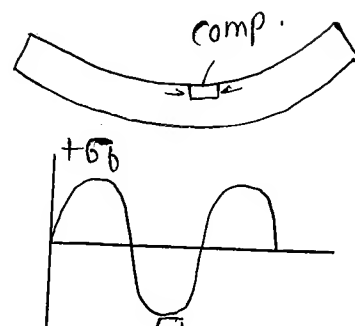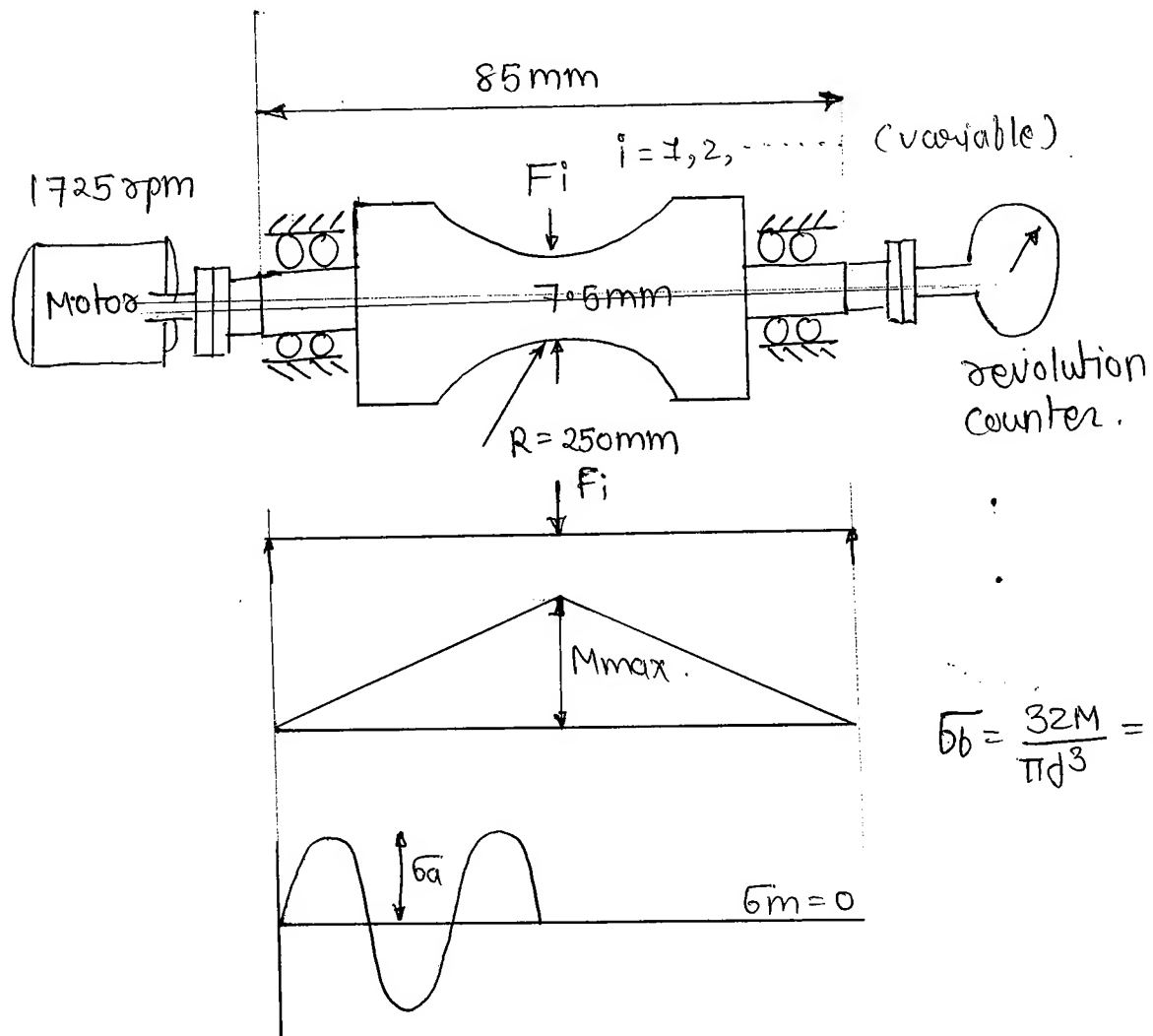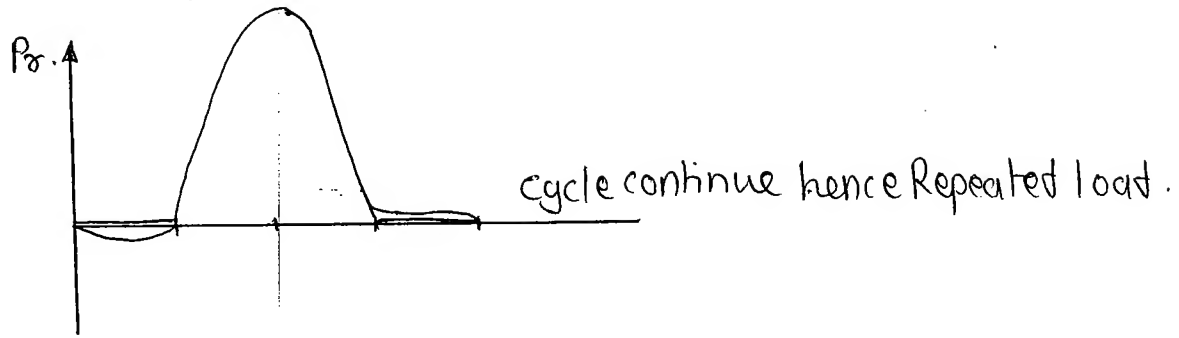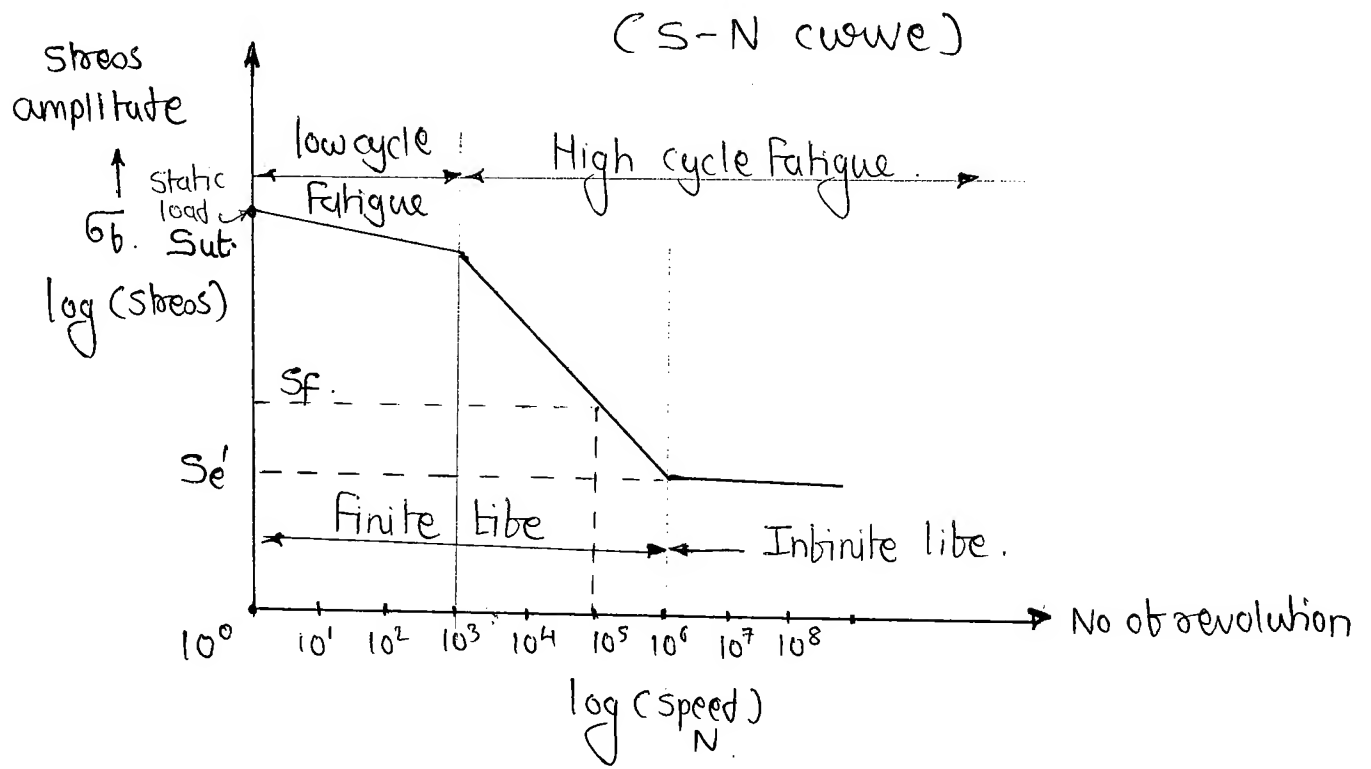
Endurance strength (Se')

It is the maximum value of a completely reversible bending stress which a standard polished specimen can withstand for a a minimum life of one million revolution.

Stress of Finite life expressed as Fatigue strength
Stress of infinite life expressed as endurance limit.

low cycle

Sut

Se'

Non Ferrous melels
Al3, cu etc

knee

Finite life ←→ Infinite life.

$10^3 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8$
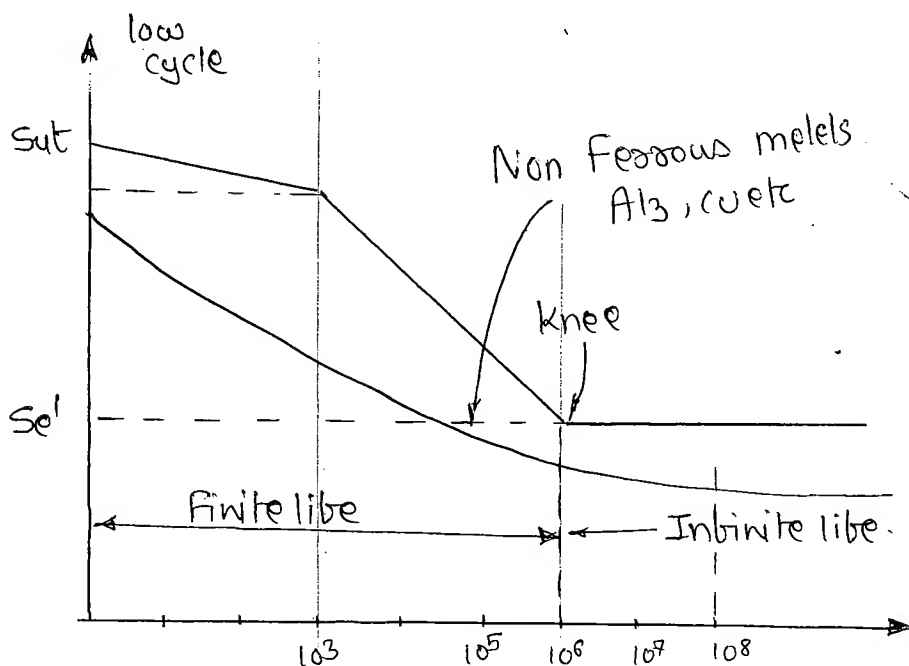
10^6 rev   — 9.6 hr.

10^8 rev   — 9.6×100

$$= \frac{960 \text{ hr}}{24} = 40.2 \text{ days}$$

$$= 1\frac{1}{2} \text{ months}.$$

Empirical relation

For steel –
$$S_e' = 0.5 \, S_{ut} \quad \text{of} \quad S_{ut} < 1400 \text{ MPa}$$
$$= 700 \text{ MPa} \quad \text{if} \quad S_{ut} \geqslant 1400 \text{Pa}.$$

For CI
$$S_e' = 0.4 \, S_{ut}$$

For non ferrous materials.
$$S_e' = 0.3 \, S_{ut}$$

Factors affecting the edurence strength of Component

Corrected endurence strength

$$S_e = k_a k_b k_c \, k_a \, k_e \, k_f \cdots \cdots S_e'$$

1) Surface finish factor (k_a)

$k_a = 1$ for polised specimen.
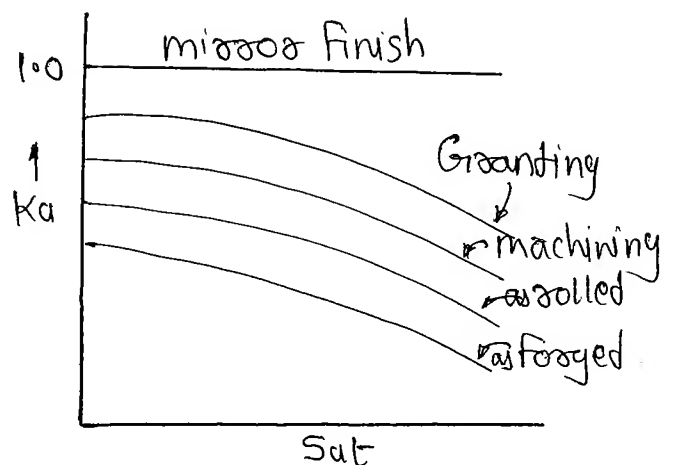
2] K_b = Size Factor

$K_b = 1$ $d \leq 8mm$

   $= 1.189 \, d^{-0.097}$

As dia of component increases the value of size factor reduces

of K_b ↓

3] K_e = Reliability Factor

   $= 1$ 50% reliability

   $= 0.897$ 90%   "

4] k_e = load factor.

   $= 1$.   Completely reversed bending. $\left(\boxed{}\right) \pm M$
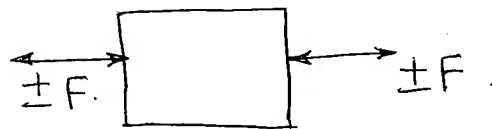
   $= 0.7$ to 0.8 (If two values limit is given & select list one)

Ex:- Connecting rod
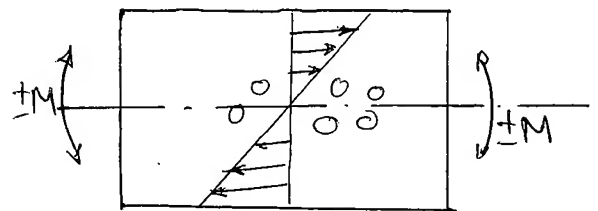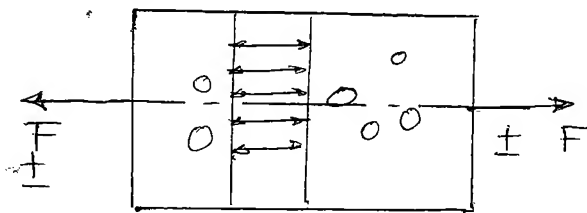
5] k_g = Temperature factor

   $= 1$ $T < 300°C$

   $= 0.5$ $T > 300°C$

$K_d = \underline{\text{Modifying stress Concentration Factor}}$

$$= \frac{1}{K_f}$$

K_f = Actual stress concentration factor

Stress Concentration

Localised high stress near the discontinuity is called Stress concentration

$\sigma \, uniform = F$

$$= \frac{F}{w \times t}$$
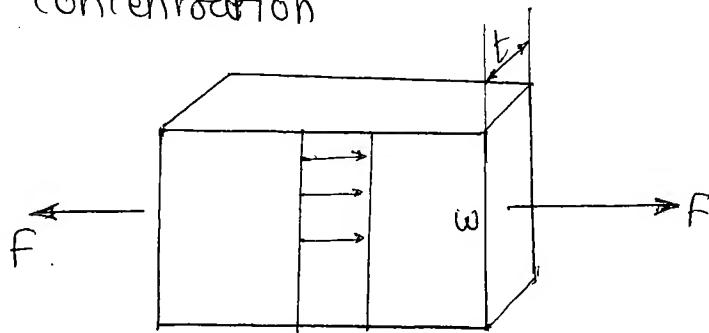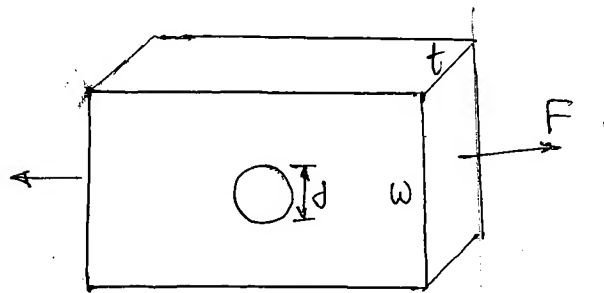
$$\sigma_0 = \frac{F}{(w-d)t}$$
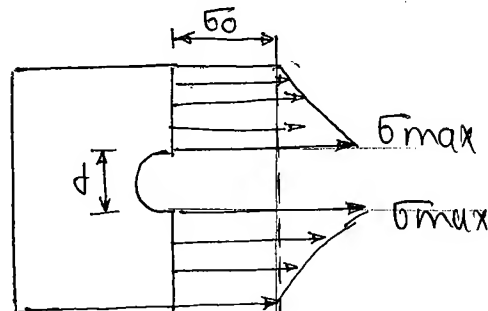
Theoratical stress concentration Factor,

$$K_t = \frac{\text{Maximum stress near the discontinuity}}{\text{Nominal stress obtained by min area of c/s}}.$$

$$K_t = \frac{\sigma max}{\sigma_0}$$

$\sigma_0 = \sigma uniform$ $(d << w)$

Infinite plate

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right) \qquad (\sigma_0 = \sigma_\theta)$$

i] If $a = 0$.

$\sigma_{max} = \sigma$

ii] If $b = 0$

$\sigma_{max} = \infty$
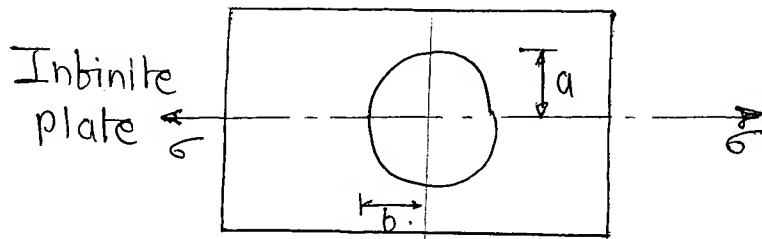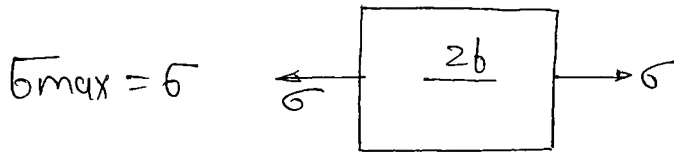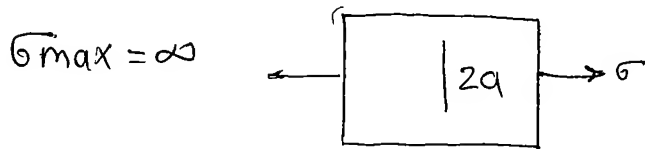
iii] If $a = b$. A circular hole

$\sigma_{max} = 3\sigma$.
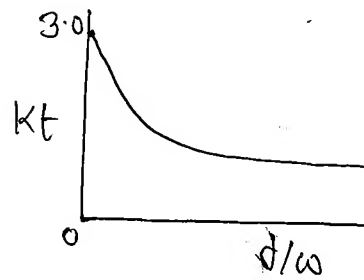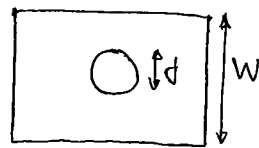
Methods of determining, K_t.

1] Methamatical analysis
        theory of elasticity

2] Experimental method
  - a] Photo-elasticity
    b] Brittle coating
    c] Strain gauge method.
    d] Grid method.
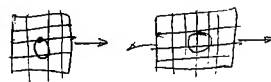
3] Computer based

Notch Sensitivity Factor (q)

The degree to which the effect of theoratical stress concentration is actually reached

$$q = \dfrac{\text{Increase of actual stress over nominal stress}}{\text{Increase theoratical stress over nominal stress}}$$

$$K_f = \dfrac{\sigma_{max}\,actual}{\sigma_0}$$

$\sigma_{max}\,actual = K_f\,\sigma_0$

Increase $= K_f\,\sigma_0 - \sigma_0$

$$K_t = \dfrac{\sigma_{max}}{\sigma_0}$$
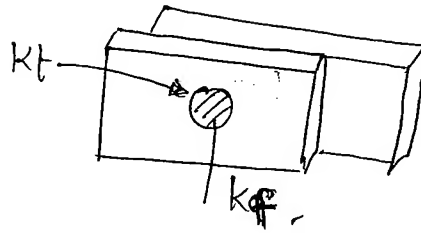
$\sigma_{max} = K_t\,\sigma_0$

Increased $= K_t\,\sigma_0 - \sigma_0$

$$q = \dfrac{K_f\,\sigma_0 - \sigma_0}{K_t\,\sigma_0 - \sigma_0} = \dfrac{(K_f - 1)\,\sigma_0}{(K_t - 1)\,\sigma_0}$$

$$= \dfrac{K_f - 1}{K_t - 1}$$

$$\boxed{K_f = 1 + q\,(K_t - 1)} \qquad q \le 1.$$

If $q = 1$

$$\boxed{K_f = K_t}$$
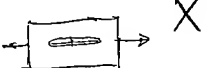
① Effect of K_t on type of Material

Type.	Static load	Fluctuating load
Ductile Material	⊡→ X	✓ K_t
Brittle Material	✓ K_t	✓ K_t

Ductile material.

$S_{yt} = 300$ MPa.

150

150

at 50 MPa

$\sigma_{max} = \sigma \left(1 + \dfrac{2a}{b}\right)$.

100 MPa

300

150=6

100

at 100 MPa, 150.

As $\sigma \uparrow$ $\sigma_{max} \downarrow$
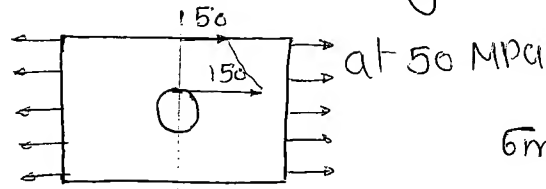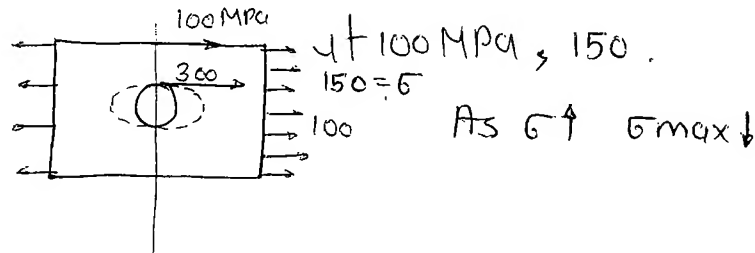
Effect of Mean stress on endurance limit.

$\sigma_a = 0$

σ_m

σ_m
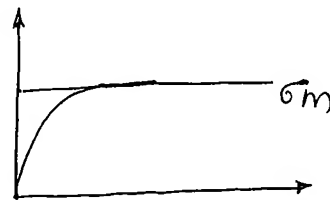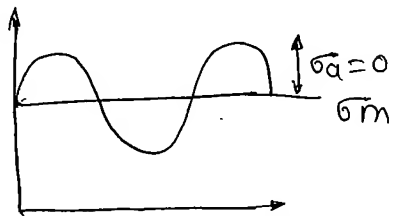
we have considered
completely reverse
condition

$\sigma_a = 0$ static load.

Torsion

Air

stac
load

$\downarrow \pm M$

stac/mean

σ σ +   Completely reversed

$\pm M$

$\pm M$

σ

stress

No of cycle $\sigma_m = 0$ Fluxituahing

σ_a

$+$ $=$ σ_m

Ex:- ① a load Normal (Bending)
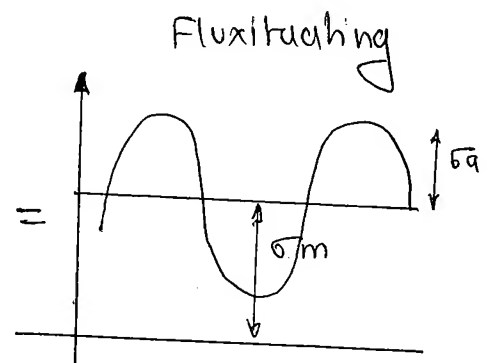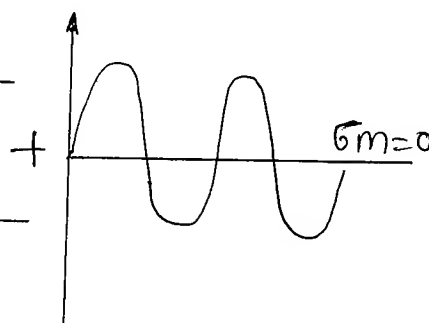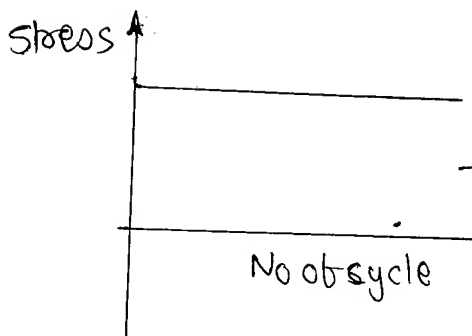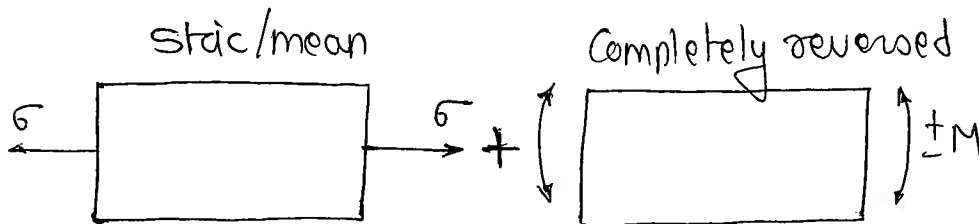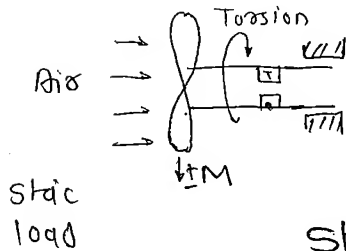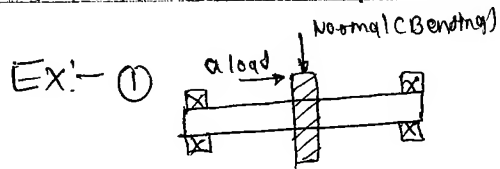
②. IC Engine connecting rod

③ Preloaded ~~load~~ boalt ~~of~~ IC engine steam engine

④ Cycle spokes.

$$\sigma max = \sigma a + \sigma b$$
$$\sigma min = \sigma a - \sigma b$$
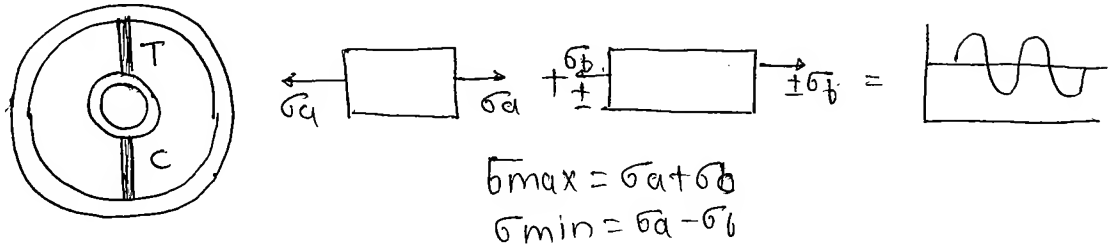
Effect of mean Stress on Endurance strength.
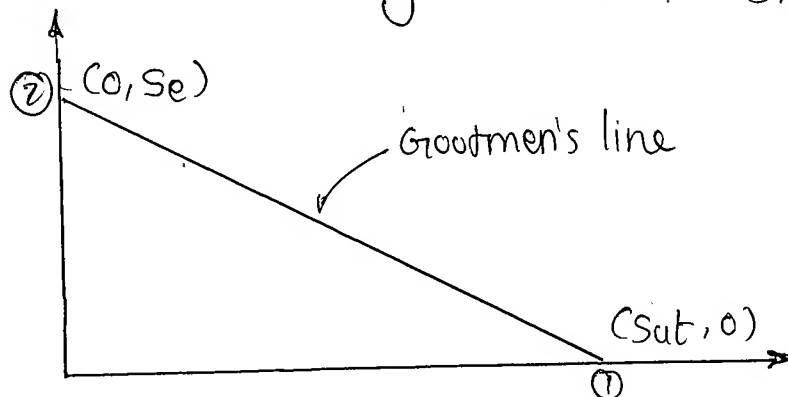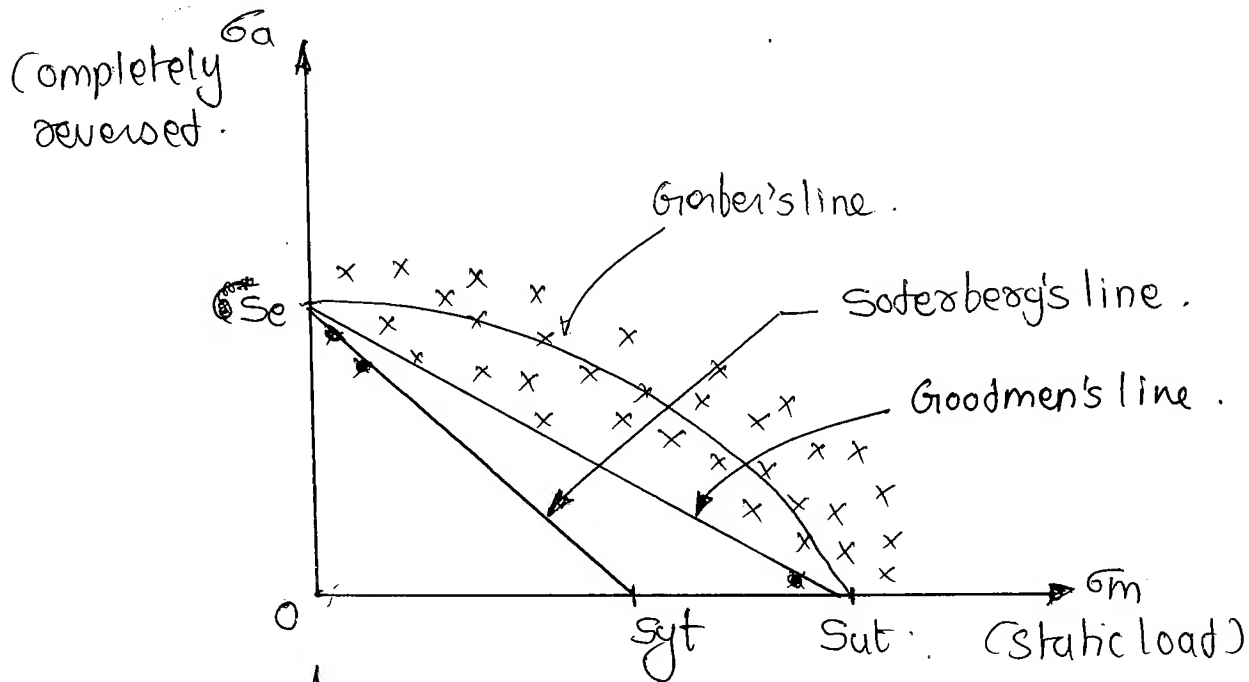
$$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1)$$

$$\sigma a - 0 = \frac{0 - Se}{Sut - 0}(\sigma m - Sut)$$

$$\frac{\sigma_a}{\sigma_{me}} = -\frac{\sigma_m}{S_{ut}} + \frac{S_{ut}}{S_{ut}}$$

$$\frac{\sigma_a}{\frac{S_e}{F \cdot S}} + \frac{\sigma_m}{\frac{S_{ut}}{F \cdot S}} = 1 .$$

$$\boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{F \cdot S}}$$

2] Soderberg's equations.

$$\boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{F \cdot S}}$$

3] Gerber's equation.

$$\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = 1 .$$

$$\boxed{\frac{\sigma_a}{S_e/F \cdot S} + \left(\frac{\sigma_m}{S_{ut}/F \cdot S}\right)^2 = 1}$$

Q :— A steel plate shown in fig is subjected to completely reversed load of 16 KN . The plate is made of steel (S_{ut} = 440 MPa) , $q = 0.8$, $k_a = 0.67$, $k_b = 0.85$, $k_c = 0.9$ $k_d = 0.897$, $k_t = 2.35$ and F·S = 1.5
a) Then the plate thickness for infinite life is

a] 6mm   b] 12mm   c] 20mm   d] 30mm

b] Optimum thickness is obtaind by

a] Gerber's Method
b] Goodmen eq^h
c] Soderber's eq^h
d] Both Goodmen & Soderbergs method
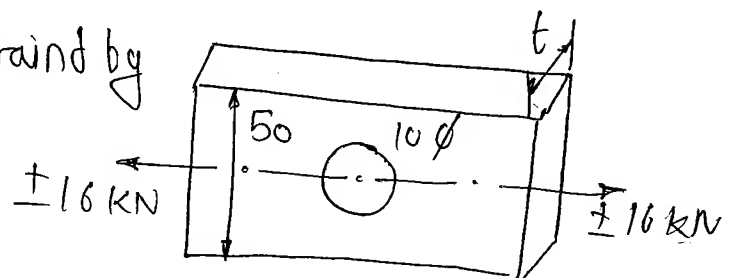
$$\frac{\sigma a}{Se} + \frac{\cancel{\sigma m}^{\,0}}{\cancel{Sut}} = \frac{1}{Fs} \qquad \text{Good men}$$

$$\frac{\sigma a}{Se} + \frac{\cancel{\sigma m}^{\,0}}{\cancel{Syt}} = \frac{1}{Fs} \qquad \text{Soderberge.}$$

Ans d]

$$\frac{\sigma a}{Se} = \frac{1}{F \cdot S}$$

$F = \pm 16\,kw$

$Sut = 440\,MPa$

$q = 0.8$

$Ka = 0.67$

$Kb = 0.85$

$Kc = 0.9$

$Kd = 0.897$

$t = ?$

$Fs = \cancel{1.5}\,1.5$

$Kt = 2.35 \div$ It given then

$$Se = ka\, kb\, Kc\, ke \cdots Se'$$
$$= 0.67 \times 0.85 \times 0.9 \times 0.897 \times \frac{1}{2.08} \times 220$$
$$= 48.62\,MPa.$$

$$Se' = 0.5\,Sut$$
$$= \underline{440 \times 0.5} \quad \text{if} < 1400\,MPa.$$
$$= 220\,MPa$$

ke — Modifing stress concentration fuctor.

$$ke = \frac{1}{Kf} = \frac{1}{1+q(Kt-1)}$$
$$= \frac{1}{1+0.8(2.35-1)} = \frac{1}{2.08}.$$

$$\sigma a = \frac{16 \times 10^3}{(w-d)t} = \frac{16 \times 10^3}{(50-10)t} =$$

$$\frac{\sigma a}{Se} = \frac{1}{F \cdot S}$$

$$\frac{16 \times 10^3}{40 \times t} = \frac{48.62}{\cancel{F}\,1.5}$$

$$\boxed{t = 12\,mm}$$

Compressive strength of material
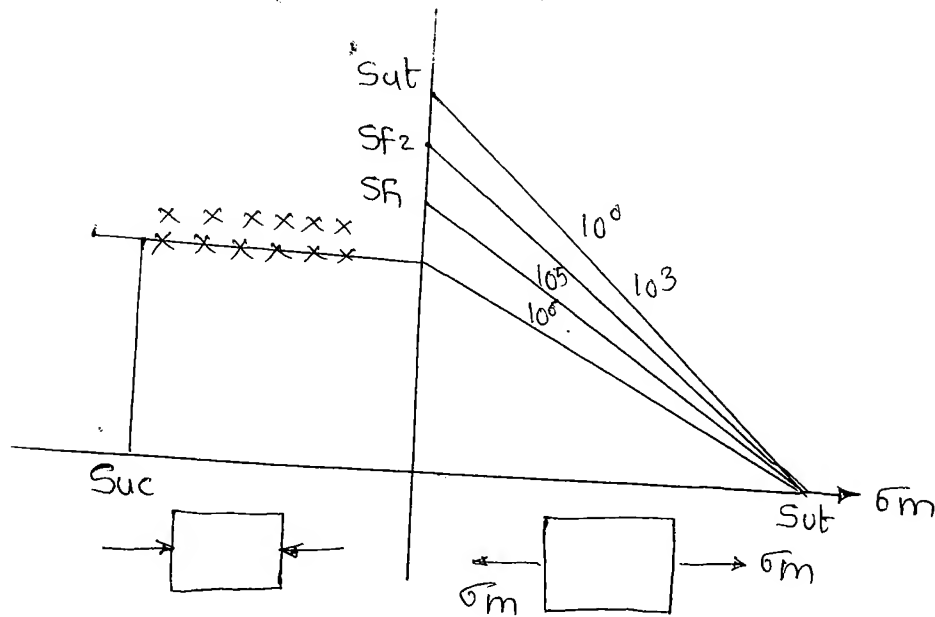
To improve fatigue strength

1] Cold rolling
2] Burnishing
3] Shot pining.
4] Case hardning
5] Coining.

} Residual stress or Preloading

Shot pining

Nozzle
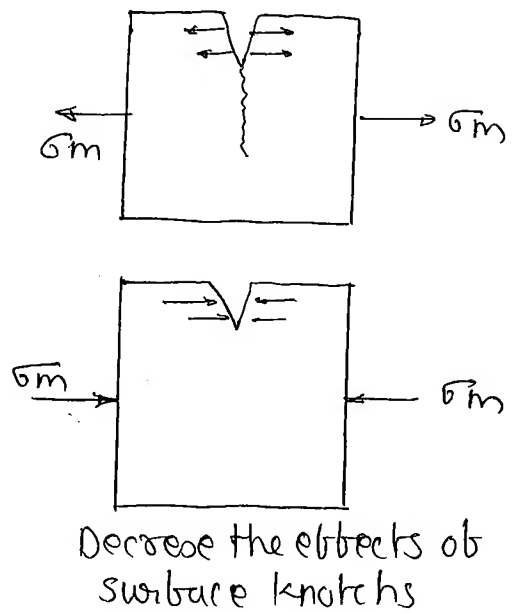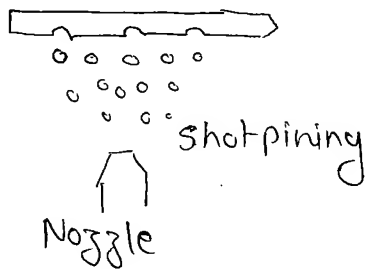
Becoz. of intention the strength is increased in case of sand cushing surface scratches produce which retures surface fatigue strength.

Decrese the effects of surface knotchs
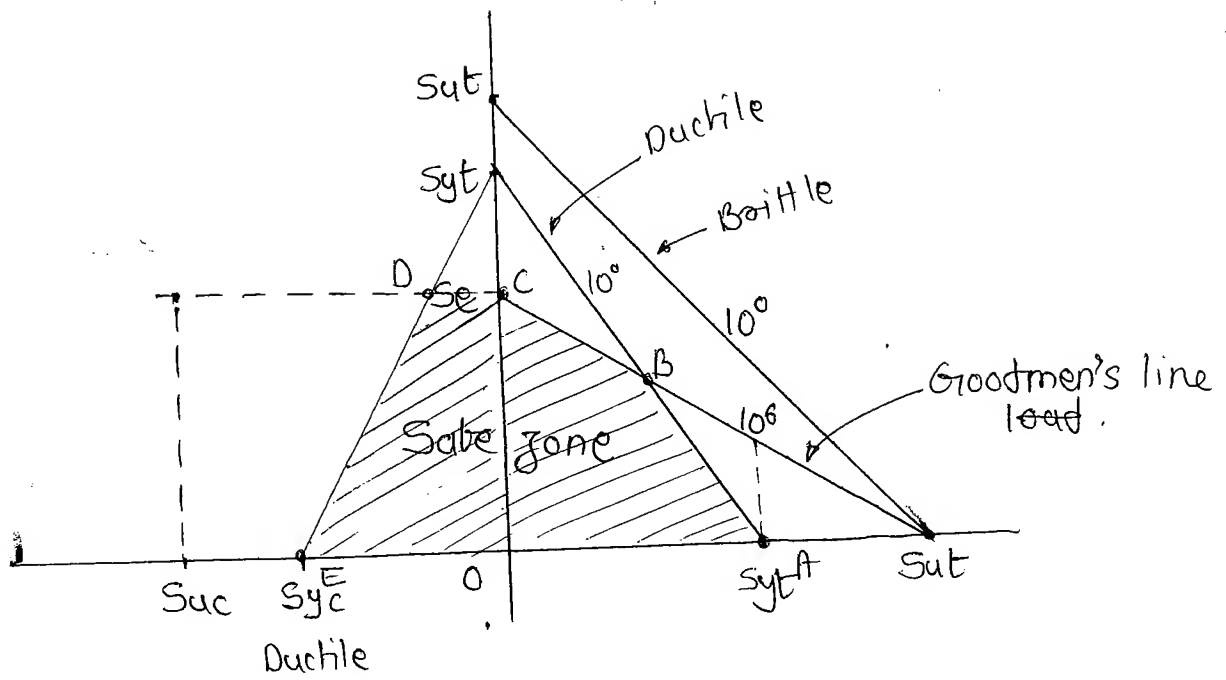
Assuming material
will Fail at syt line
hence give name 10°

Design for finite life.
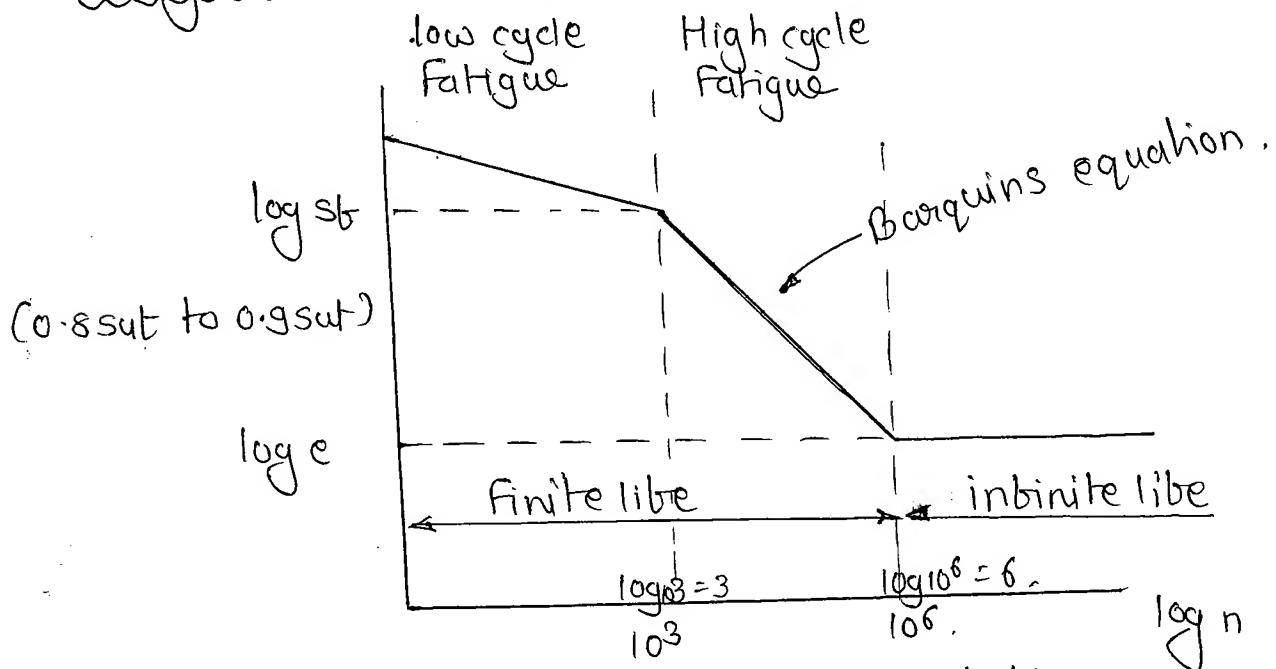
Design for finite life and high cycle fatigue

Sf corresponding to 10^3 rev is 0.8 sut to 0.9 sut.

Basquin's Equations.

$$A = S_f L^B.$$

A and B are constants.

S_f - fatigue strength

L - Life in revolution.

$$A = S_e (10^6)^B \quad -①$$
$$A = S_f (10^3)^B \quad -②$$

Find the value of A & B then, using above ① & ② eqⁿ.

Cumulative fatigue.

Stress
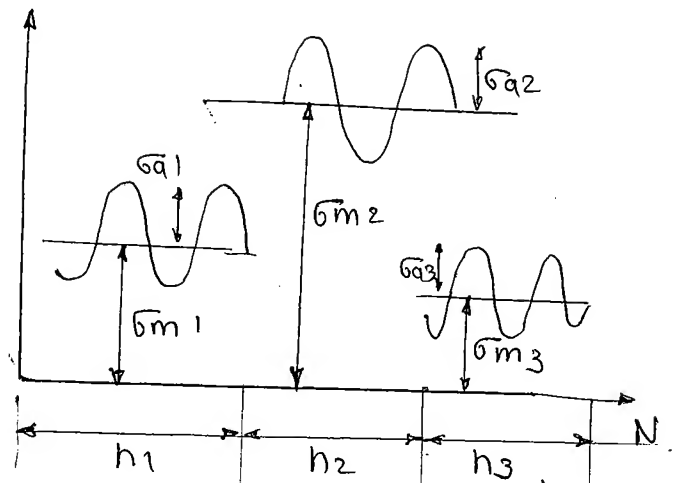
Ex:- Pulley, Bearings

Cumulative fatigue - Miner's equation

Let, a component is subjected to

Stress	σ_1	for	n_1	no of cycle.
	σ_2	"	n_2	"
	σ_3	"	n_3	"
	σ_4	"	n_4	"

If the component is subjected to only stress σ_1 then, lets its life $= L_1$ revolution

Life completed in one revolution $= \dfrac{1}{L_1}$ ← total life

Life completed during n_1 revolution $= \dfrac{n_1}{L_1}$

Total life consumed $= \dfrac{n_1}{L_1} + \dfrac{n_2}{L_2} + \dfrac{h_3}{n_3} + \cdots \cdots + \dfrac{n_x}{L_x} = 1$

Let, $n_1 = \alpha_1 L$

$n_2 = \alpha_2 L$

$L \to$ total life
$\alpha_i \to$ part of the total life at G_i

$\dfrac{\alpha_1}{L_1} + \dfrac{\alpha_2}{L_2} + \cdots \cdots + \dfrac{\alpha_x}{L_x} = \dfrac{1}{L}$

Q 19/20 A machine part is made of steel with $Sut = 630\,MPa$, $Se = 0.22\,Sut$, Fatigue strength for 10^3 cycles is $0.9\,Sut$ The life of part is 25000 cycles The load for 50% of time is $\pm 225\,Mpa$ and 30% of time is $\pm 145\,MPa$.

The lifes of part at $225\,MPa$ and $145\,MPa$ respectively are (in revolution)

a] 9.29×10^4, 8.01×10^5

c] 8×10^4, 4×10^5

b] 9.7×10^4, 6×10^5

d] 3×10^5, 5×10^5.

Ans $Sut = 630\,Mpa$ $L = 16\,25000$
$Se = 0.22\,Sut$ $Sf = 10^3$ cycles is $0.9\,Sut$.

$$A = Sf \, L^B.$$

$Sf_1 = \pm 225\,MPa$ $\alpha_1 = 0.5$
$Sf_2 = \pm 145\,MPa$. $\alpha_2 = 0.3$.

$Se = 0.22 \times 630$ $= 138.6$ $\cdots \cdots$ 10^6 rev
$Sf = 0.9 \times 630$ $= 567$ $\cdots$ 10^3 rev.

$$A = 138.6 \times 10^{6\,B}$$
$$A = 567 \times 10^{3\,B}.$$

$138.6 \times 10^{6\,B} = 567 \times 10^{3\,B}$ $10^{6B-3B} = \dfrac{5.63 \times 10^3}{138.6 \times 10^6}$

$B = 0.203$
$A = 2304$

$$230H = Sf L^{0.203}$$

$$230H = 225 L_1^{0.203}$$

$$L_1 = 9.32 \times 10^4$$

$$L_2 = 8.03 \times 10^5$$

The load during the remaining time.

a) 250 MPa   b) 375 MPa   c) 395 MPa   d) 415 MPa.

$$\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} + \frac{\alpha_3}{L_3} = \frac{1}{L}$$

$$\frac{0.5}{9.32 \times 10^4} + \frac{0.3}{8.03 \times 10^5} + \frac{0.2}{L_3} = \frac{1}{25500}$$

$$L_3 = \cancel{8756.15} \; 5837.46 \text{ cycle}.$$

$$230H = Sf \, L^{0.203}$$

$$230H = Sf \times 5837.46^{0.203}$$

$$\boxed{Sf = 396 \text{ MPa}}$$

29] The life of a machine component under heavy test load was 10 hrs. The life under combine loading was 18 hours of which 9.8 hr at heavy load and 8.2 hours was at the normal load. What is the expected life under normal operating load

a) 54.5hr   b) 6hrs   c) 410 hr   d) 490 hr.

$$\frac{n_1}{L_1} + \frac{n_2}{L_2} = 1.$$

$$\frac{9.8}{10} + \frac{8.2}{L_2} = 1$$

$$\boxed{L_2 = 410}$$

OR

$$\alpha_1 = \frac{9.8}{18} \quad , \quad \alpha_2 = \frac{8.2}{18}$$

$$\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} = \frac{1}{L} \qquad \frac{0.511}{10} + \frac{0.455}{L_2} = \frac{1}{18}$$

Welding

Welding is a process of joining metals by the application of heat with and without the application of pressure.

Advantages

1] Highest Joint efficiency
2] Lighter in weight
3] Easy to weld any part of the structure at any point.
4] Easy to make alteration

Joints $\longrightarrow$ Butt
$\searrow$ Fillet

Disadvantages

1] Due to non uniform heating and cooling thermal stresses will be induced it result in destorstion of component.

Reto Reinforcement
$(kt = 1.2)$
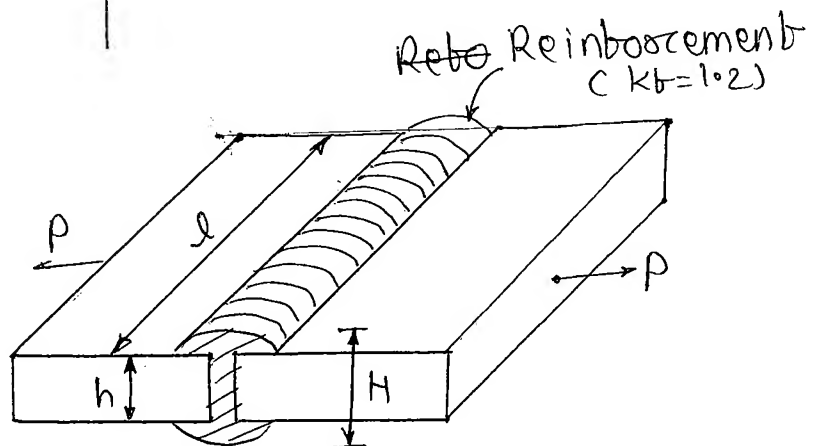
$$\sigma t = \frac{P}{h \times l}$$
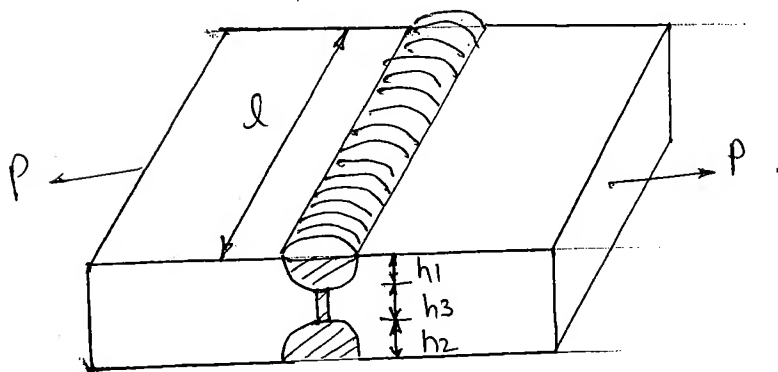
$$\sigma t = \frac{P}{(h_1 + h_2) l}$$

Fillet weld.
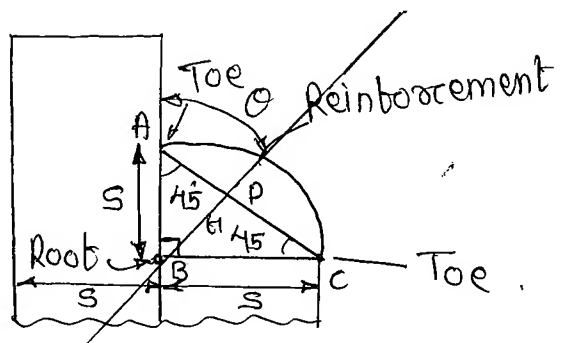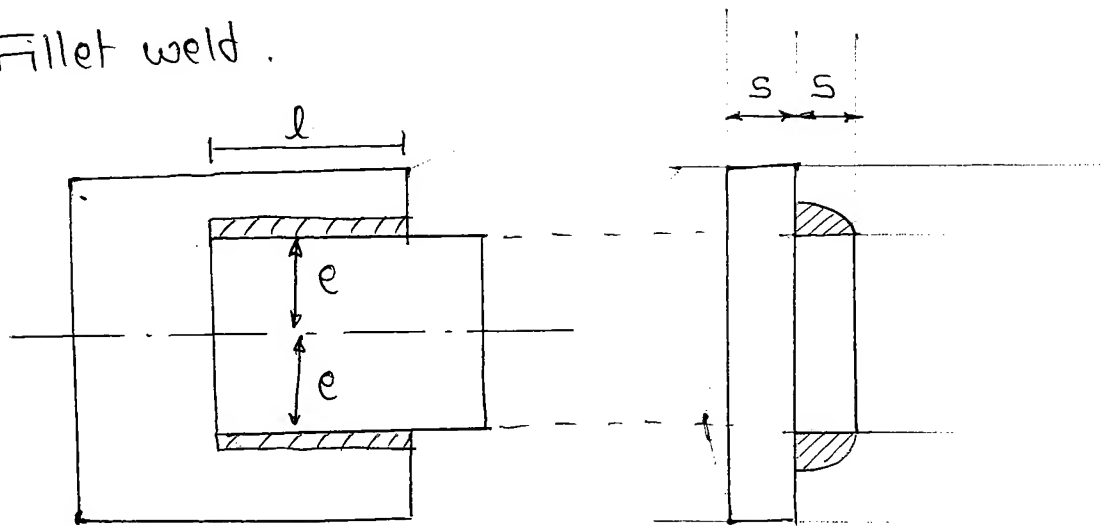
Toe and Root are the weakest point because of high stress concentration

$$\frac{t_1}{\sin 45} = \frac{s}{\sin(180 - (45 + \theta))}$$

$$t_1 = \frac{S \sin 45}{\sin(45 + \theta)}$$

$$= \frac{S \sin 45}{\sin 45 \cos\theta + \cos 45 \sin\theta}$$

$$= \frac{S}{\cos\theta + \sin\theta}$$
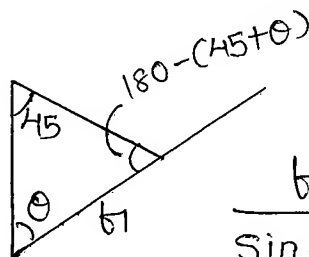
$$t_1 = \frac{S}{\sin\theta + \cos\theta}$$

Shear stress on the plane at an angle θ

$$\tau_\theta = \frac{P}{t \times l}$$

$$\tau_\theta = \frac{P(\sin\theta + \cos\theta)}{S \times l}$$

Max^m shear stress

$$\frac{\partial \tau}{\partial \theta} = 0$$

$$\frac{P}{Sl} \neq 0.$$

$$\frac{\partial}{\partial \theta}(\sin\theta + \cos\theta) = 0$$

$$\cos\theta - \sin\theta = 0$$

$$\cos\theta = \sin\theta = 45^\circ.$$

$$t = \frac{S}{\sin\theta + \cos\theta} \qquad \theta = 45.$$

$$\boxed{t = 0.707 \, S} \quad - \text{ throat thickness}.$$

$$\tau_{45^\circ} = \tau_{max} = \frac{P(\sin 45 + \cos 45)}{Sl}.$$

$$\tau_{max} = \frac{\sqrt{2} \, P}{Sl}$$

$$\boxed{P = \frac{1}{\sqrt{2}} Sl \, \tau_{max}}.$$

Load carrying capacity of single parallel fillet weld

$$P = 0.707 \, Sl \left(\frac{S_{sy}}{F \cdot S} \right) \; -$$

$$P = 1.404 \, Sl \left(\frac{S_{sy}}{F \cdot S} \right) - \text{Double fillet weld.}$$

$$\tau_{90} = \frac{P}{Sl} \qquad \frac{\tau_{90}}{\tau_{45}} = 0.707$$

$$\tau_0 = \frac{P}{Sl}$$

Transverse fillet weld.

$\tau_{max} = \tau_{67.5}$

shear

$\to$ P .

$\to$ P
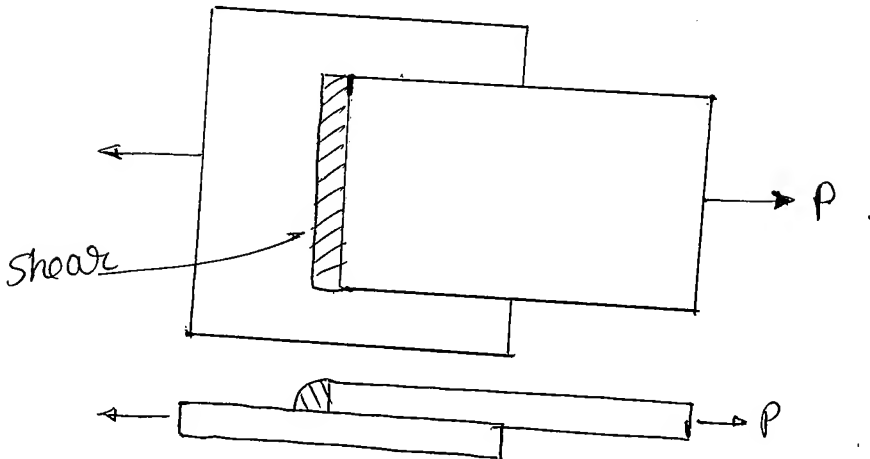
$\tau_{max} = \tau_{67.5}$

$P = 0.707 \, Sl \, \tau_{max}$ — Transverse fillet weld

$$\frac{\tau_{67.5}}{\tau_{45}} < 1 .$$

H =

$H = P \sin\theta$

$N = P \cos\theta$
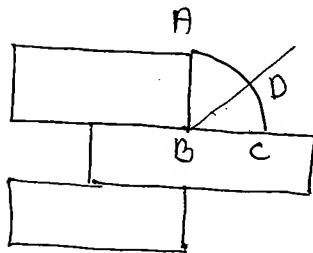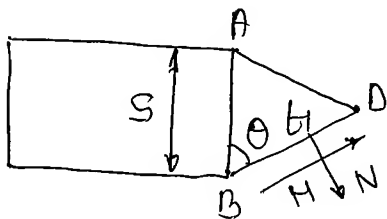
$$\tau = \frac{H}{t_1 l}$$

$$= \frac{P\sin\theta \,(\sin\theta + \cos\theta)}{Sl} .$$

$$\frac{\partial \tau}{\partial \theta} = 0$$

$$2\theta = 135° .$$

Axially loaded unsymmetrical welds.
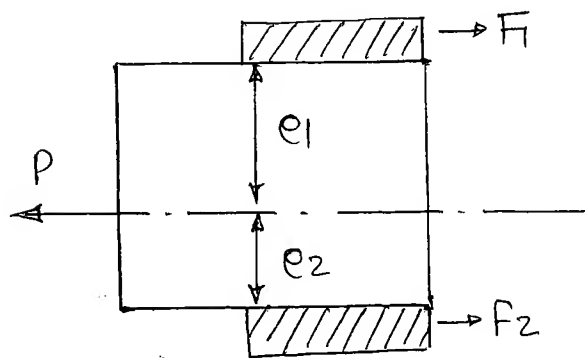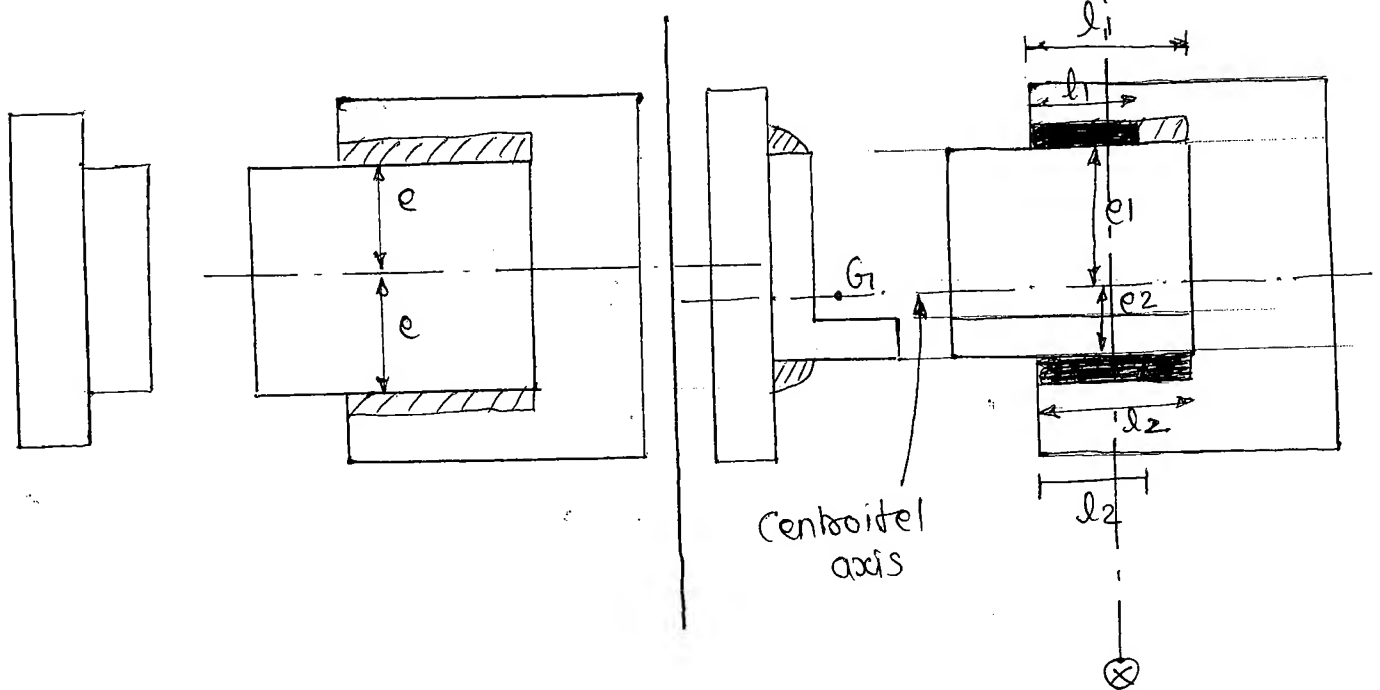
Centroidal axis

$$P = F_1 + F_2$$

$$\Sigma M_{\text{centroidal axis}} = 0.$$

$$F_1 \times e_1 - F_2 \times e_2 = 0.$$

$$F_1 e_1 = F_2 e_2$$

$$\frac{F_1}{e_2} = \frac{F_2}{e_2} = \frac{F_1 + F_2}{e_1 + e_2} = \frac{P}{e_1 + e_2}$$

$$F_1 = \frac{e_2}{e_1 + e_2} \times P.$$

$$F_2 = \frac{e_1}{e_1 + e_2} P_2$$

Ex:-

$$P = 0.707 \, Sl \left(\frac{Ssy}{F \cdot S} \right)$$

$$l = l_1 + l_2$$

$$F_1 = 0.707 \, S \times l_1 \left(\frac{Ssy}{F \cdot S} \right)$$

$$F_2 = 0.707 \, S \times l_2 \left(\frac{Ssy}{fs} \right)$$

$$P = F_1 + F_2$$

$$l = l_1 + l_2$$

$$P \propto l$$
$$F_1 \propto l_1$$
$$F_2 \propto l_2$$

$$F_1 = \frac{e_2}{e_1 + e_2} \, P.$$

$$l_1 = \frac{e_2}{e_1 + e_2} \times l$$

$$l_2 = \frac{e_1}{e_1 + e_2} \times l.$$
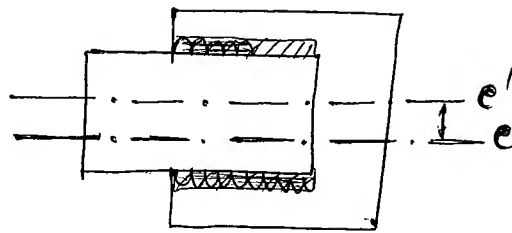
Bending movement will take place because of change in centroidle axis.

Eccentrically loaded Welted Joint.
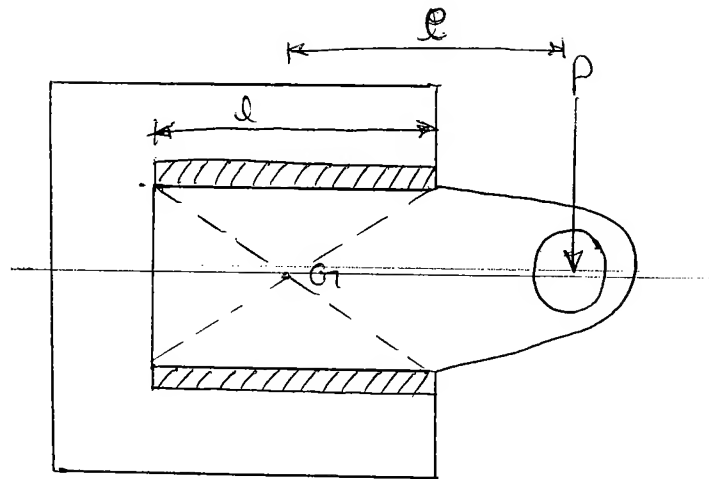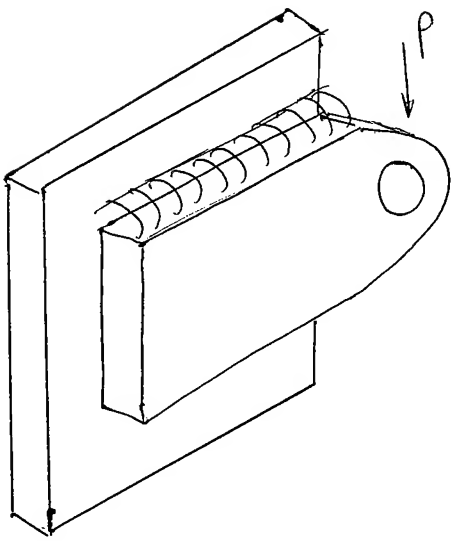
eccentic load cause a Movement
Stress it may be bending or
torsion according to the application
of load

$M_x = P \times \ell = T.$

$M_y = 0$

$M_z = P \times \ell$
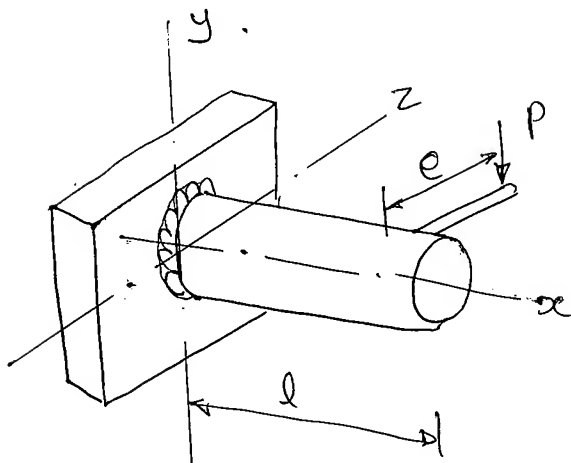
$M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$

$T + O + B$

Any member trying to move member about its own
axis is called Torsion.

If $\ell = 0$. there is only torque.

$t << \ell$

$t << e$
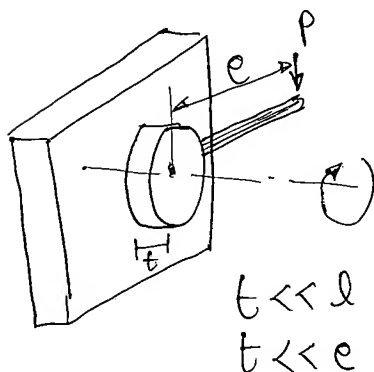
Hence it causes a torsional
movement.

$$\tau_1 = \frac{P}{2t \times \ell}$$

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{T \times r}{J}$$

$$\tau \propto r$$
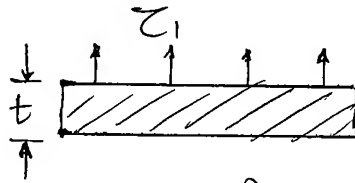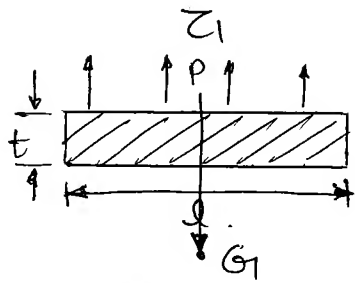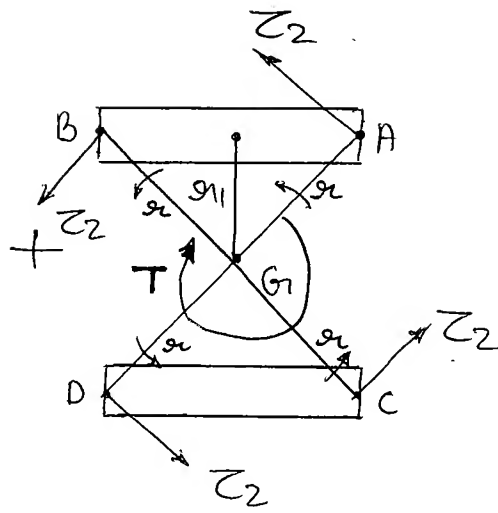
$$+ \quad \tau_2 = \frac{T}{J} \times r$$
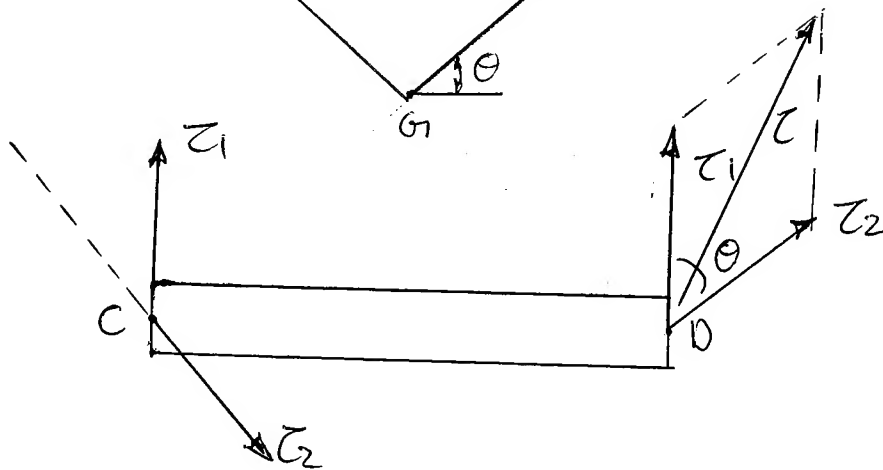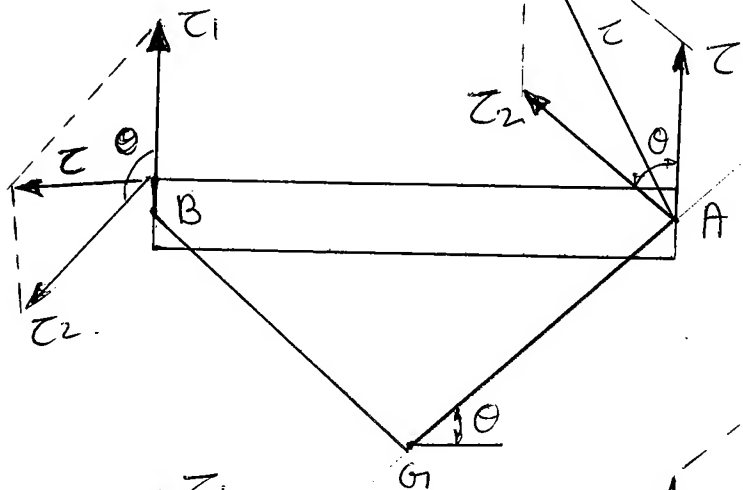
$$\tau = \sqrt{\tau_1^2 + \tau_2^2 + 2\tau_1\tau_2 \cos\theta}$$

$$\tau = \phi(\tau_1, \tau_2, \theta)$$

$$\theta \downarrow \quad \cos\theta \uparrow$$

Hence object will first fail from point A & D

Polar movement of Inertia.
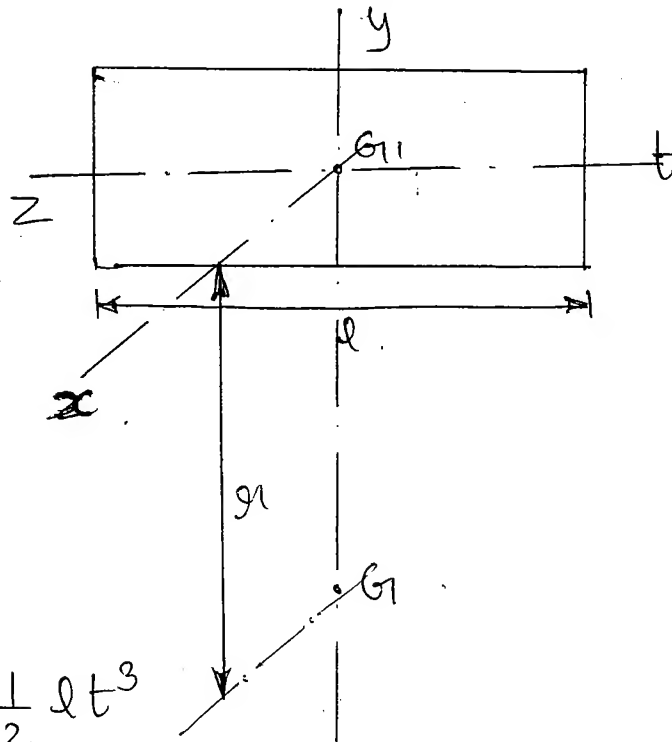
$$I_z = \frac{1}{12} \ell t^3$$

$$I_y = \frac{1}{12} t \ell^3$$

$$I_x = J_{G_1} = I_z + I_y$$

$$= \frac{1}{12} \ell t^3 + \frac{1}{12} t \ell^3 \qquad t \lll \ell .$$

$$\boxed{I_{G_1} = \frac{1}{12} t \ell^3 .}$$

$$I_G = I_{G_1} + A r^2$$

$$= \frac{1}{12} t \ell^3 + t \times \ell \cdot r^2 .$$

$$\boxed{I_G = A \left[\frac{\ell^2}{12} + r^2 \right]}$$

For both site

$$\boxed{I_G = 2A \left[\frac{\ell^2}{12} + r^2 \right]}$$

Note:- It two thickness value is given then take the lowest value for weld design.

1] The permissible shear stress in a fillet used is 100 MPa. The fillet has equal leg length of 15mm each. The allowable shear load on the weldment per mm is

a] 2.25 KN   b] 1.15 KN   c] 1.06 KN   d] 0.15 KN

$S = Size = leg$

$S = size = leg$
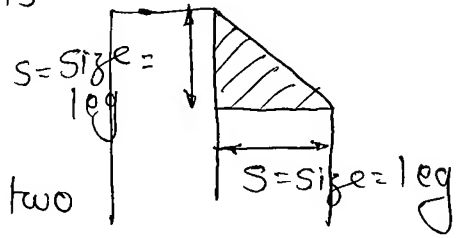
Size of weld = least thickness of two plates

$$P = 0.707 \, Sl \left(\frac{S_{sy}}{FS} \right)$$

$$= 0.707 \times 15 \times 1 \times 100$$

$$= 1.06 \, KN.$$

2] The allowable load per linear centimeter of the fillet weld for the 10mm size, assume shear stress intensity to be 80 MPa is

a] 2.8 KN   b] 5.6 KN   c] 8 KN   d] 11.3 KN.

$$P = 0.707 \, Sl \left(\frac{S_{sy}}{FS} \right)$$

$$= 0.707 \times 10 \times 10 \times 80$$

$$= 5.6 \, KN.$$

Estimate the shear stress due to Torsion (T)

$$Z = \frac{T}{J} \times r$$

$t = 0.7078$

θ — shear stress

T, Z

M

$I_Z = \pi t r^3$

$I_y = \pi t r^3$

$$I_x = \int_{\theta=0}^{\theta=2\pi} y^2 \, dA$$

$y = r\sin\theta$

$dA = r \, d\theta \, t$

$$= \int_{\theta=0}^{\theta=2\pi} r^2 \sin\theta \times r \, d\theta \times t$$
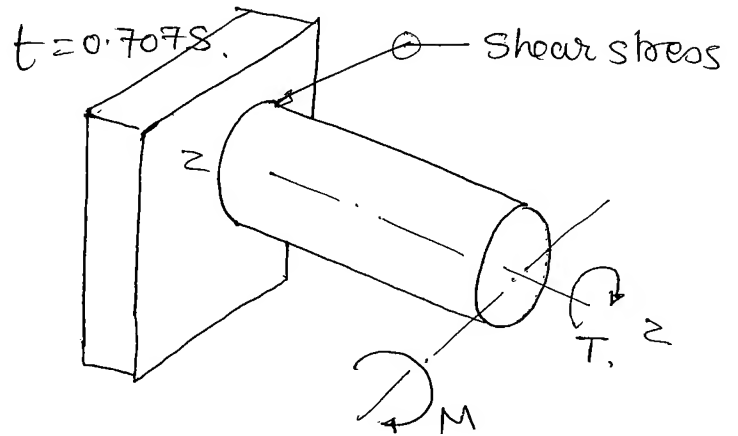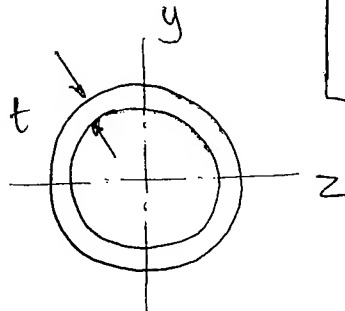
$$= r^3 t \int_{\theta=0}^{\theta=2\pi} \left(\frac{1-\cos 2\theta}{2}\right) dt$$

$$= \frac{r^3 t}{2} \int 1$$

$$= r^3 t \, \pi$$

$$= \pi t r^3$$

$$I_y = \pi t r^3$$

$$J = I_x = I_z + I_y = 2\pi t r^3$$

$$Z = \frac{T}{J} \times r$$

$$= \frac{T}{2\pi t r^3} \times r = \frac{T}{2\pi t r^2} \qquad \boxed{\tau = \frac{T}{2\pi t r^2}}$$
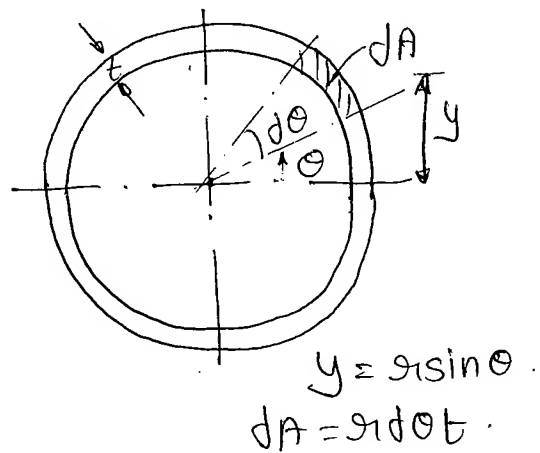
Bending stress (σ_b) due to B.M

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M}{I} \times y$$

$$= \frac{M}{\pi t \, r^3} \times r$$

$$\boxed{\sigma_b = \frac{M}{\pi t \, r^2}}$$

Ratio beth the Bending and Shear stress due to M & T.

$$\frac{\sigma_b}{\tau} = \frac{M/\pi t r^2}{T/2\pi t r^2}$$

$$\frac{\sigma_b}{\tau} = \frac{M}{T/2}.$$

$$\boxed{\frac{\sigma_b}{\tau} = \frac{2M}{T}}$$

Q:- The moment on the circular billet weld whose axis coinside with x axis is given by.

$$M = \overset{Mx}{40 \hat{j}} + \overset{M_z}{20 \hat{k}}$$ The ratio of bending & Shear stress is

a) 2   b) 4   c) 3   d) 1

$$\frac{\sigma_b}{\tau} = \frac{2 \times 20}{40} = 1.$$

$$\frac{\sigma_b}{\tau} = \frac{2M}{T}.$$

Q   The torque T, that can be applied, if the permissible shear stress is 140 MPa, is

a] 2332.5 N-m   b] 1452.3 N-m   c] 1252 N-m

d] None.

$$\tau = \frac{T_d}{2\pi t r^2} = 2$$

$$T = \tau \cdot 2\pi t r^2 \quad \searrow^{0.707 \times 6} \quad \overline{t_{50}}$$

$$= 140 \times 2\pi \times t \times 25^2$$

$$= 2332.5 \, N-m$$
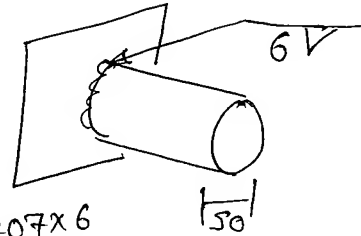
Q25 — The fillet weld weld of length 100mm each are used to support flat 'b' on the bracket 'a' Flat 'b' carries only tensile load. The leg length of both fillet weld is 5mm. The properties of weld metal are $S_{ut} = 480 \, MPa$, $S_{yt} = 410 \, MPa$, $S_{sy} = 230 \, MPa$ and reduction in area 40%. Find the load P.

a] 230 kN   b] 162.6 kN   c] 480 kN   d] 80.8 kN

$$t = 0.707 \, S = 0.707 \times 5$$

$$X \, P = 2 \times l \times t \times S_{yt} \times 1.4. \quad P = 0.707 \times S \times l \left(\frac{S_y}{FS}\right) \times 2$$

$$= 0.707 \times 5 \times 100 \, (230) \times 2$$

$$= 162.61 \times 10^3$$

$$= 162.61 \, MPa.$$
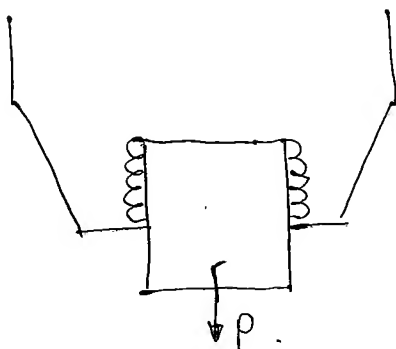
Q: A billet weld joint is subjected to transverse loading 'F' as shown in fig. Both legs of the billets are of 10 mm size and the weld depth is 30mm. If the allowable shear stress of the weld metal is 94 mpa. Considering the minimum throat area of the weld, the maximum allowable transverse load in kN is
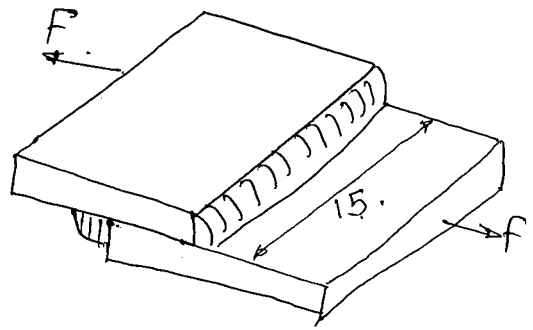
A] 14·44

b] 17·92

c] 19·93

d] 22·16

$$P = l \times 0.707 \times S \times \tau.$$
$$= 0.707 \times 10 \times 30 \times 940$$
$$P = 19.93 \times 10^3 \, N.$$
$$= 19.93 \, kN.$$

Q) 26 & 27 An angle plate is welded as shown in fig. The allowable stress for weld is 75 N/mm²

26] The length of weld at top in mm is

a] 103·7   ⓑ 96·3   ⓒ 90   ⓓ 110

27] The force carried by top weld is

a] 55KN   ⓑ 60KN   ⓒ 65KN   ⓓ 50KN

$$l_1 = \frac{e_2}{e_1 + e_2} \, l.$$
$$l_1 = \frac{55}{155 + 45} \, \text{for} \, (l_1 + l_2)$$
$$l_1 = \frac{5.11}{40} \, l.$$

$$l_2 = \frac{145}{200} \, (l)$$
$$l_2 = \frac{29}{40} \, l.$$
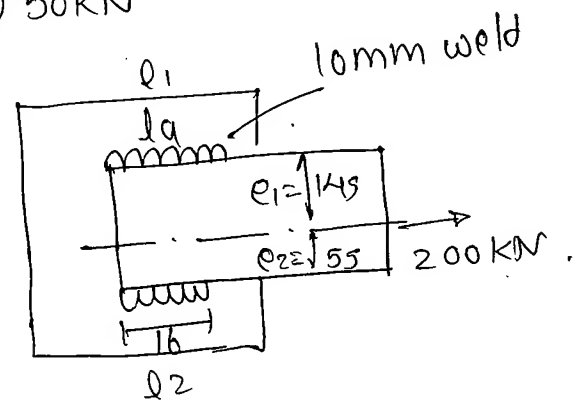
$$F_1 = \frac{e_2}{e_1 + e_2} \, P.$$
$$= \frac{55}{200} \times 200$$
$$= 55 \, kN.$$

$$F_1 = 0.707 \, S \, l_1 \left(\frac{S_{sy}}{F_s} \right)$$
$$55 = 0.707 \times 10 \, l_1 (75)$$
$$= 0.103 \, m$$
$$l_1 = 103.72$$

48]

Q 30. In the welded joint shown in fig. If the weld B has thicker fillets than at A then the load carrying capacity P of the joint will be.

a] Increases
b] Decreases.
c] unaffected
d] exactly got ~~hot~~ double
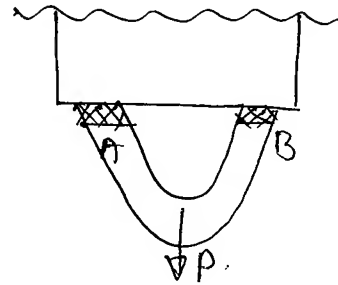
Pg 53

Q 10] Compound fillet welding joint is loaded by P as shown. The maximum shearing ~~co~~ stress accure at a point

a] A and B
b] B & D
c] only A
d] only c
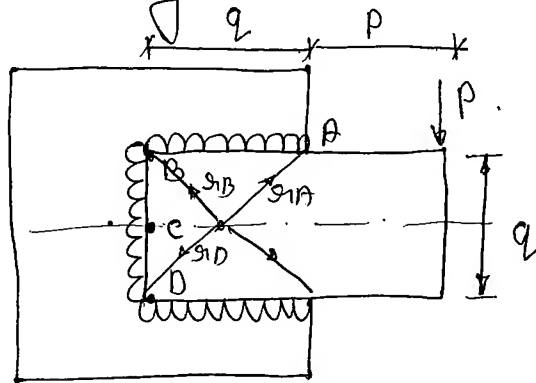
⑫ A spherical pressure vessel made of 10mm steel plate hemispheres butt welded together and the tank is to be 15m diameter. The tensile strength of weld metal is 85 MPa. The allowable internal pressure inside the tank is

a] 0.227 MPa
b] 0.452 MPa
c] 0.161 MPa
d] 0.752 MPa .

$$\sigma_h = \frac{PD}{4t}$$

$$85 = \frac{P \times 15 \times 10^3}{4 \times 10}$$

$$\boxed{P = 0.226 \ MPa}$$

Screwed Fasteners.

Designation

Md × P

Metric

Nominal
dia

Pitch

Screw Forms

① BSW

55°

② BA

47 ½

③. ANS

60

④ Isometric.

Clearance

H/8

H/4

dc   d
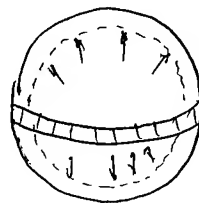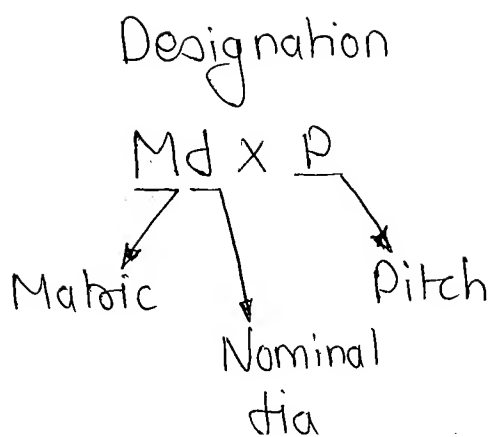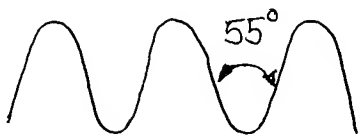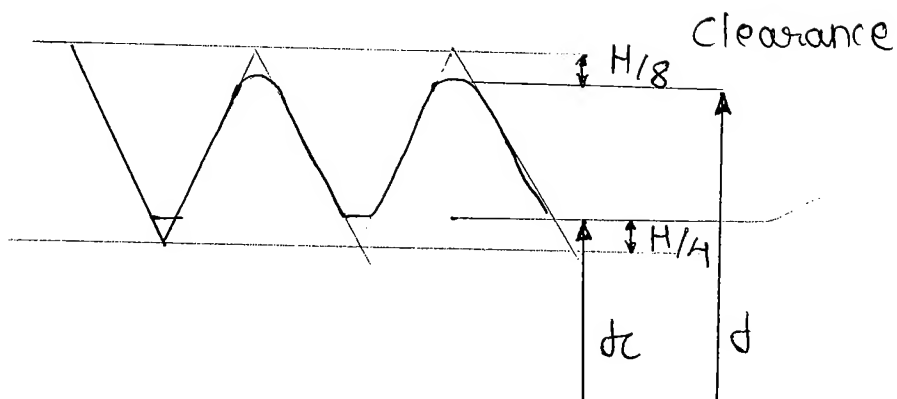
⑤ Square thread angle = 0°

P

P/2

⑥. ACME Thread

P

29°

P/2

⑦

P

45°

3/4 P
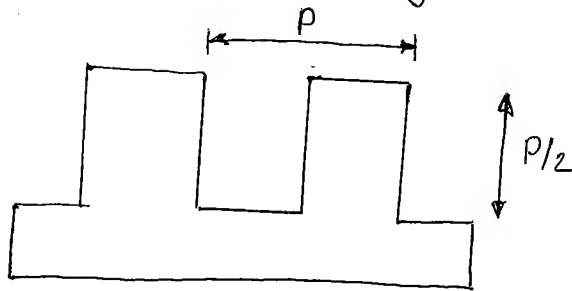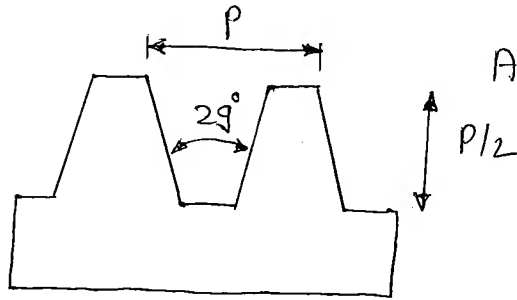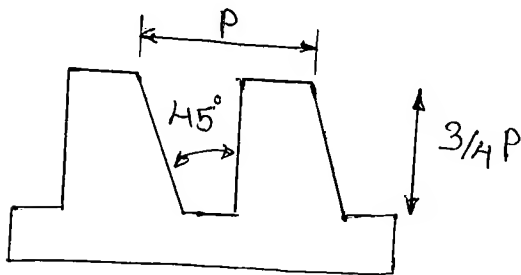
Bolted Joint   Power Transmission.

Fcosβ Power Force

F

2β

β

F sinβ
Bursting
Force

Screw Axis.
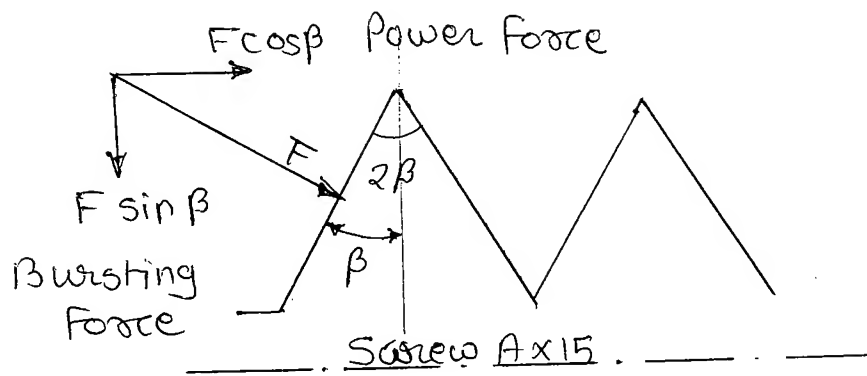
Locking force/Bursting Force
$= F \sin\beta$

$\beta\uparrow$ $F\sin\beta \updownarrow\uparrow$

Power Force
$= F\cos\beta$

$\beta\downarrow$ Power$\uparrow$

for Max power $\beta = 0$. (square thread)

Stresses in screw.

2d

d

dc

Shank

Thread.

0.8d

Represent
the nut
side.

Nut

d c

T

Bolt

SCREW

T
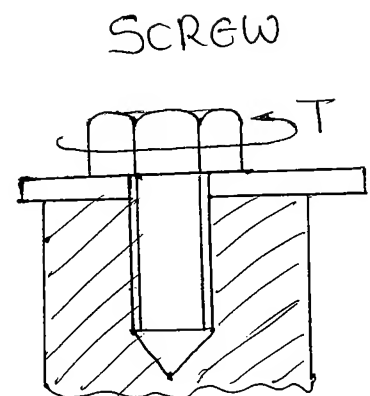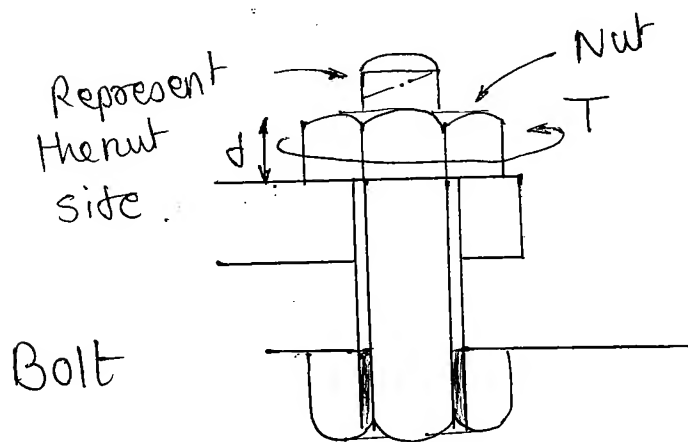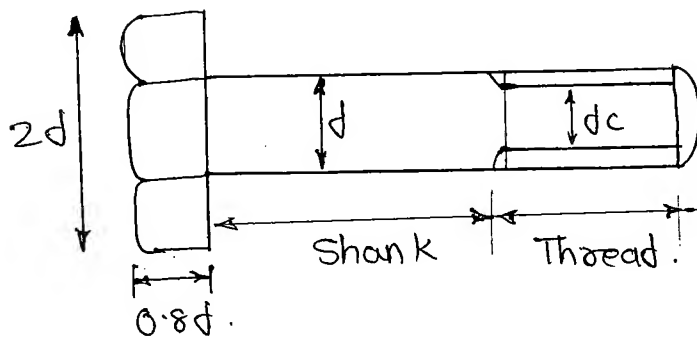
→ Bolt is alway having a nuts & torque given to nut

→ Screw is made for taped hole. It don't have nut and torque given to head

Stresses in Screw

Tightning load

I·C Engine

Shear

Wall Brackets.

tension (try to separate the pipe)
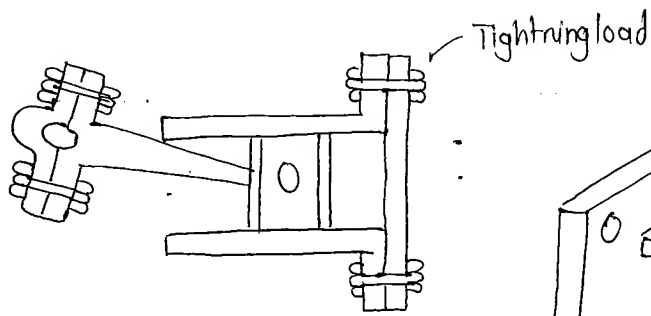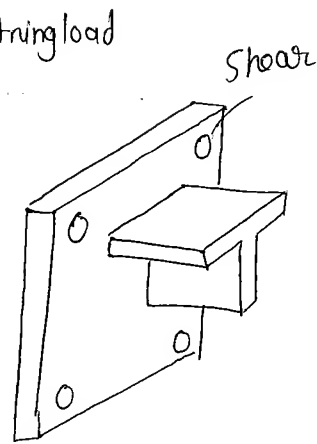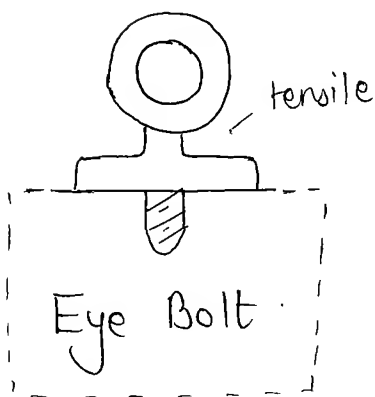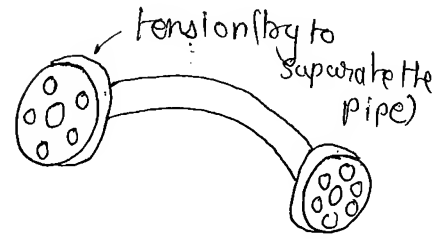
tensile

Eye Bolt
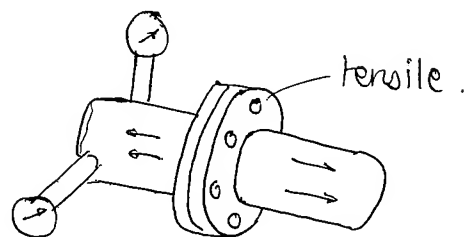
tensile.

Power vessel

We are designing the ~~load~~ Bolt for tensile force.
① Preload (To tight the bolts)
② External force (load to which bolt is subjected)

Stresses in Screw

① Stresses due to preload / Initial tightining
② Stresses due to external load
③ Stresses due to combine preload and external load.

Stresses due to Pre load.

Bolt — Fi

Connected members.

Head — Gasketed — Flange
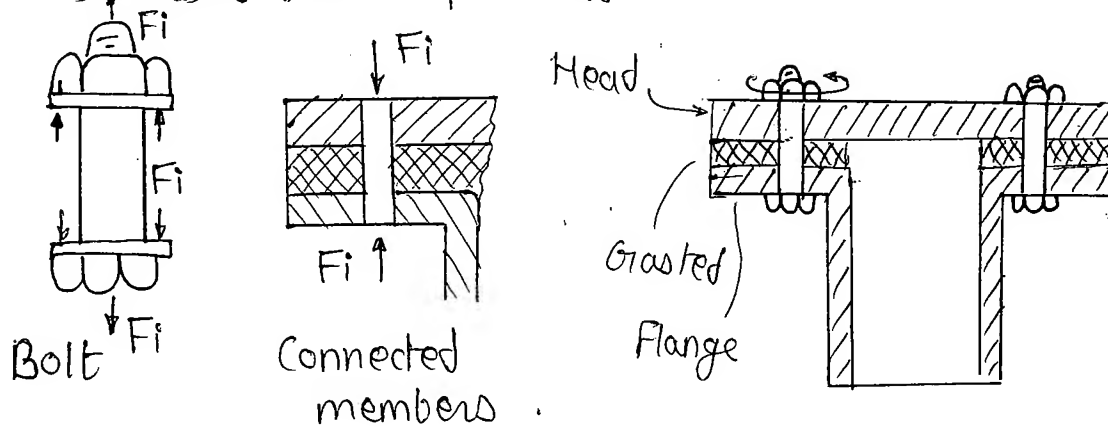
$F_i = 2840 d$ N For leak proof joint based on the Proof strength (without permanent deformation)

$F_i = 1420 d$ N For ordinary joint

Proof strength

σ

0.2%

ε
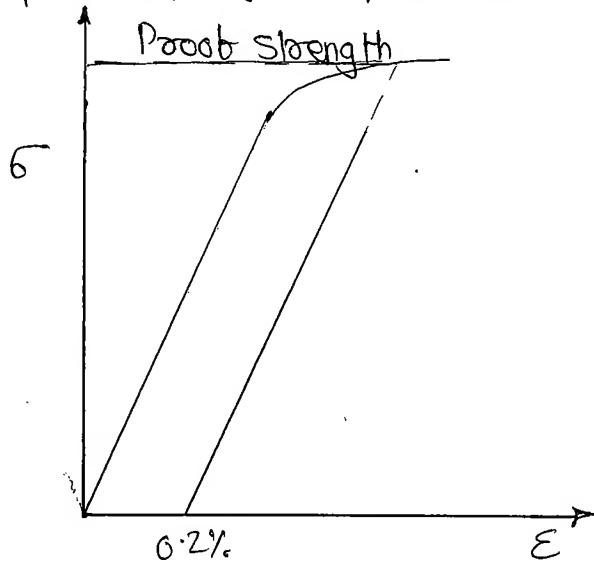
Initial load is expressed in term of dia of bolt or in term of load applied

$$F_i = 2840 d \quad or \quad F_i = 1.5 P .$$

Stresses due to Preload

① Tensile stresses.

$$\sigma t = \frac{Fi}{\frac{\pi}{4} dc^2}$$

②. Torsional shear stress

$$\tau = \frac{16T}{\pi d_c^3}$$

③. Shear stress accross thread.

Bolt made up of hard material

SCREW

NUT

nut made from a soft material
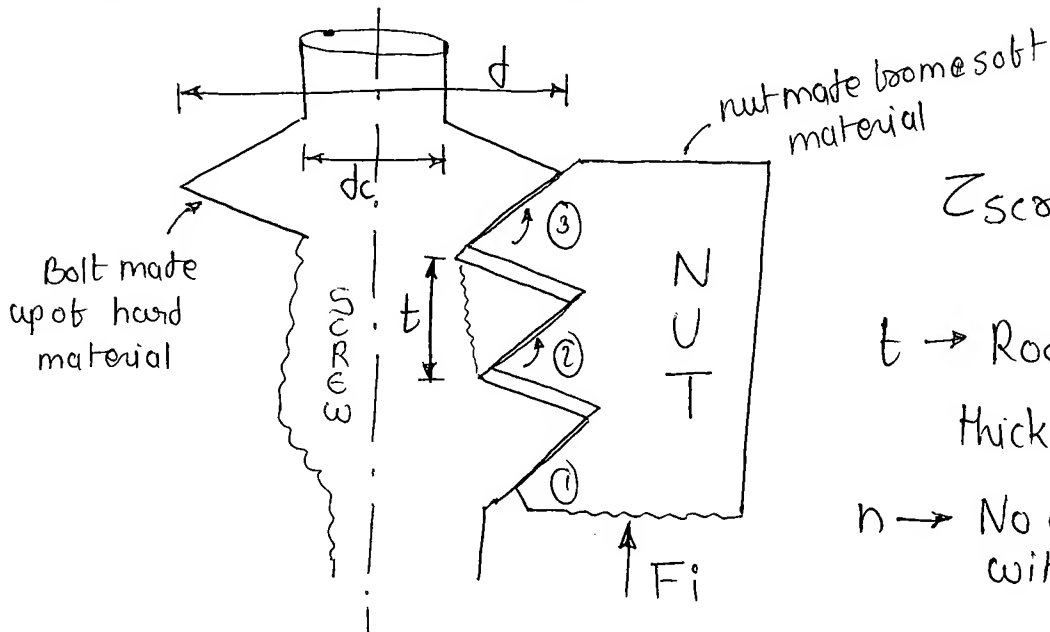
$$\tau_{screw} = \frac{Fi}{\pi dc \times t \times n}$$

$t \rightarrow$ Root thickness or thickness of thread

$n \rightarrow$ No of thread incontact with nut.

$$\tau_{nut} = \frac{Fi}{\pi d t \times n}$$

$d \rightarrow$ external dia
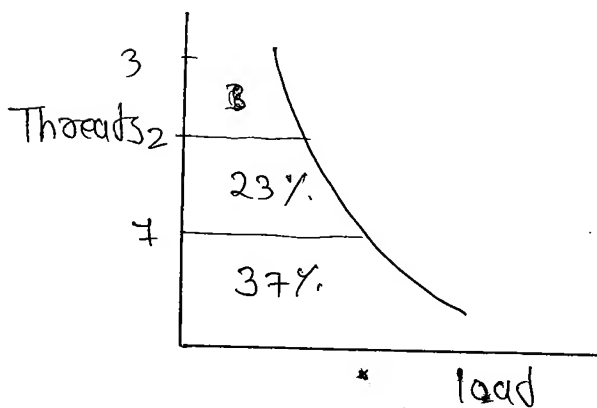
Increasing the size of nut will not give advantage because last thread take zero load

Bearing preossure — When two threads are in contact they transmits the load called as bearing pressure or crushing stress

Ⓐ Crushing stress

$$\sigma_c = \frac{Fi}{\frac{\pi}{4}(d^2 - d_c^2)\, n}$$
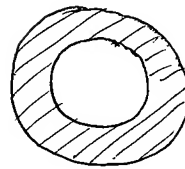
⑤ Bending stress
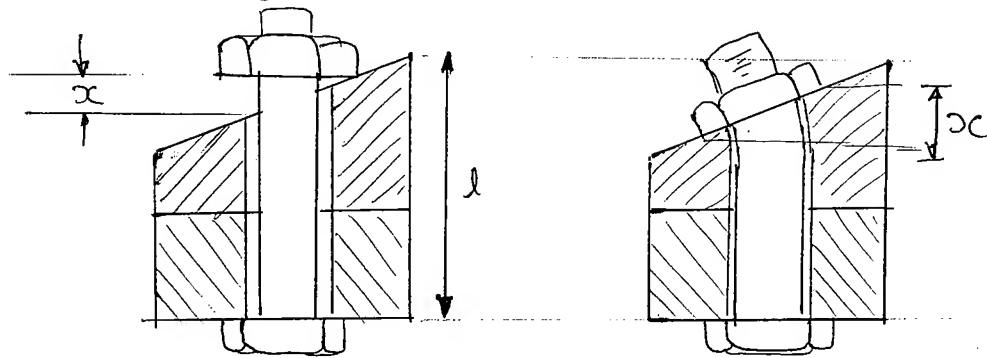
$$\sigma_b = \frac{x \times E}{2\ell\ell}$$

x — distance bet^h the external corner of nut

ℓ — holding length

E — Young's Modulus of elasticity.

Stresses due to external load.

① Tensile stress

$$\sigma_t = \frac{P}{\pi/4\, d_c^2}$$

P — External load.

Eye Bolt.
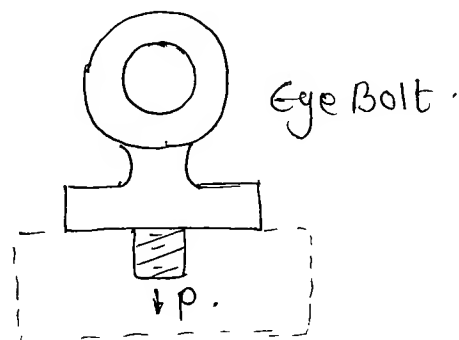
② Shear stress.

$$Z = \frac{P}{\frac{\pi}{4} d^2}$$
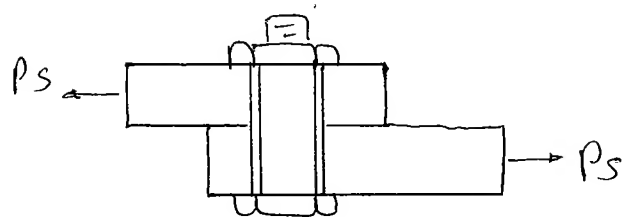
Note - Due to shear & tension combinely acting

$$\sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$$

① Guest's theory.

$$\tau_{max} = \left[\sqrt{\left(\frac{P_t}{2}\right)^2 + P_s^2}\times\right]\frac{1}{A}.$$

$$\frac{1}{A}\sqrt{\left(\frac{P_t}{2}\right)^2 + P_s^2}$$

② Distortion energy theory.

$$\sigma_e = \sqrt{\sigma^2 + 3\tau^2}$$

$$= \sqrt{\sigma_t^2 + 3\tau^2}.$$

Note:- Take d_c in both shear and tension to avoide confusions.
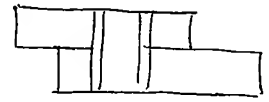
Stresses due to combined external and preloads.

Let, P_r = preosure

D = Bore dia.

External load $P = P_r \times \frac{\pi}{4} D^2$. N.

P = Total load.

$P = P_b + P_m$

$\boxed{F_i = 1.5 P}$

P_b = External load shoared by bolt

P_m = Part of external load taken by connected member.

F_b = Resultant load on bolt

$\quad = P_b + F_i$

F_m = Resultant load on connected member

$\quad = P_m - F_i$

Δ_b = Elongation of bolt due to F_i

Δ_m = Compreosion of connected member due to F_i
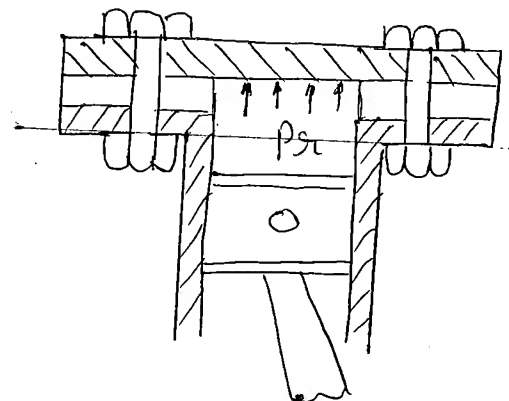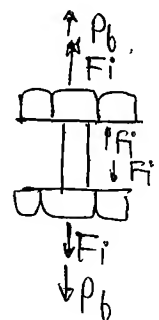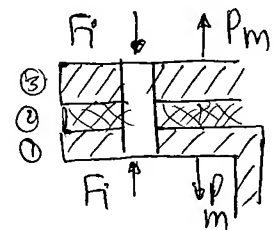
δb = $\dfrac{\text{External and Preload}}{\text{Increase in elongation of bolt due to } Pb}$.

δm = decrease in compression of conned members due to Pm

For compatability

$$\boxed{\delta b = \delta m}$$

kb = Stiffness of bolt.

$$= \dfrac{Pb}{\delta b}$$

km = Combined stiffness of connected members
(Series)

$$\dfrac{1}{km} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \dfrac{1}{k_3}.$$

$$km = \dfrac{Pm}{\delta m}$$

$$\delta b = \delta m$$

$$\dfrac{Pb}{kb} = \dfrac{Pm}{km}$$

$$Pm = \dfrac{km}{kb} \times Pb.$$

$\rightarrow$ $P = Pb + Pm$

$$P = Pb + \dfrac{km}{kb} \times Pb.$$

$$P = Pb \left[\dfrac{kb + km}{kb} \right]$$

$$Pb = \left[\dfrac{kb}{kb + km} \right] P.$$

$$Pm = \left(\dfrac{km}{kb + km} \right) P.$$

C = stiffness factor.

$$= \frac{Kb}{(Kb + Km)}$$

$$Pb = CP.$$

$$Pm = (1-C) P.$$

$$\boxed{Fb = Pb + Fi}$$

$$Fm = Pm - Fi$$

$$\boxed{Fm = (1-C)P - Fi}$$

i] Size of bolt.

$$Fb = \frac{\pi}{4} dc^2 \times \frac{Syt}{F.S}.$$

2] Leakage is prevented.

$Fm \le 0$ — Resultant load must be compression

$$(1-C)P - Fi \le 0.$$

$$\boxed{(1-C)P \le Fi}.$$

Pno. 34

Q.] The ratio of load required to open the bolt joint to the initial tightening load with stiffness of mating part is equal to four times stiffness of bolt is

a] 0.2   b] 0.8   c] 1.25   d] 2.5.

$$Km = 4 Kb.$$

$$\frac{P}{Fi}$$

$$(1-C) P - Fi = 0.$$

$$\frac{P}{Fi} = \frac{1}{1-C}$$

$$= \frac{1}{1 - 0.2}$$

$$= \frac{1}{0.8}$$

$$\frac{P}{Fi} = 1.25$$

$$C = \frac{Kb}{Kb + Km}$$

$$= \frac{Kb}{5 Kb} = \frac{1}{5} = 0.2$$

③ A bolted joint connets two members with an axial tightnening for F kN. The bolt has matric threads of 4mm pitch The torque req. to achivie this force F is 1.4 KN-mm what is the value of 'F'.

a] 0·35 KN   b] 0·7 KN   c] 1·05 KN   d] 2·2KN.

$p = 4mm$.
$T = 1·4 KN-mm$

Work done

$W = $ Force x distance $=$ Torque x θ

$F \times 4 \qquad = 1·4 KN-mm \times 2\pi$

$F = 2·2 KN$.

Bolted Joints.

Advantage of Preload.

washer

P_b

F_t

F_i

P_b

F_i

P_i

Load.

Hence it will cause fatigue Failure

$\delta b = \delta m$

$Fa' > Fa$

P_b

F_t

F_i

km

P_m

δm

P_b

F_i

Fa

kb

δb

Fa'

Compression Δm

Δb.

$\in$ Elongation

It will produce the locking effect due to less fluctuating load
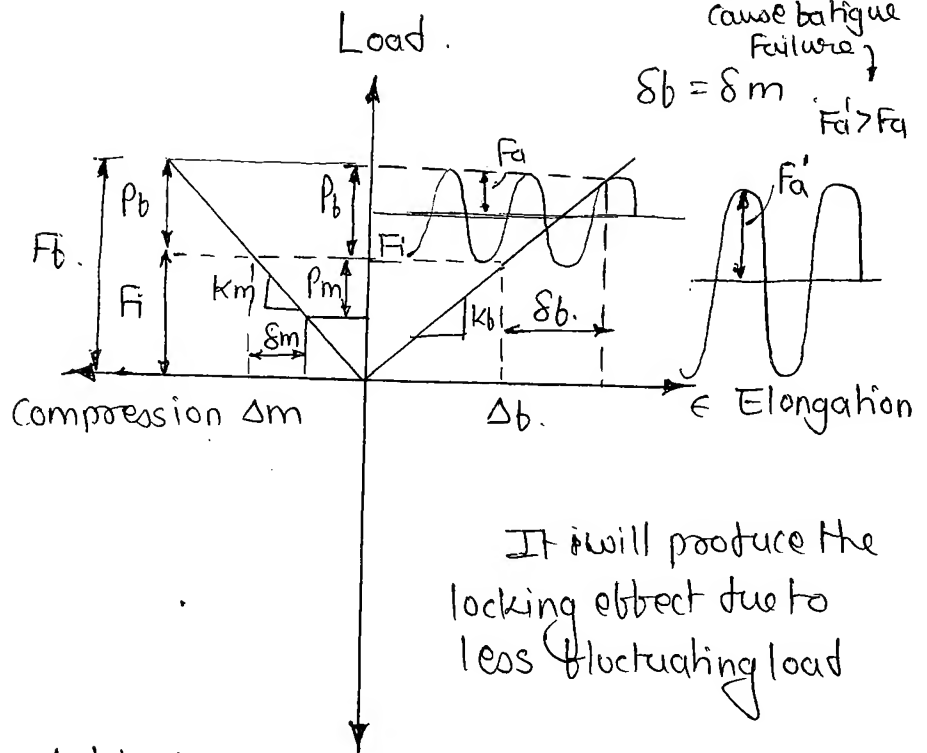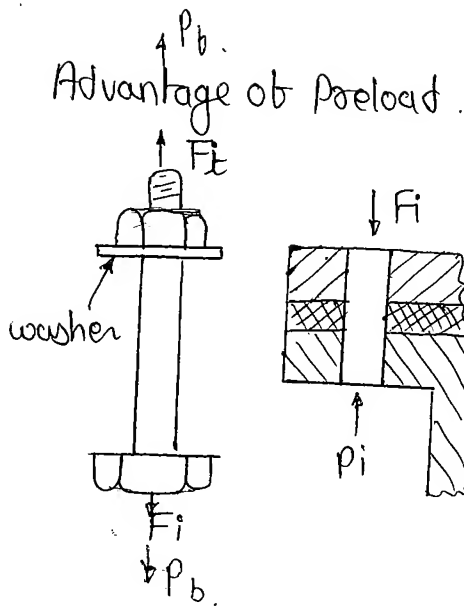
① To prevent leakage of fluid.

② Increases the fatigue strength of bolt

③ Increases the locking effect.

P_r

D.

load amplitude.

No. of cycle.
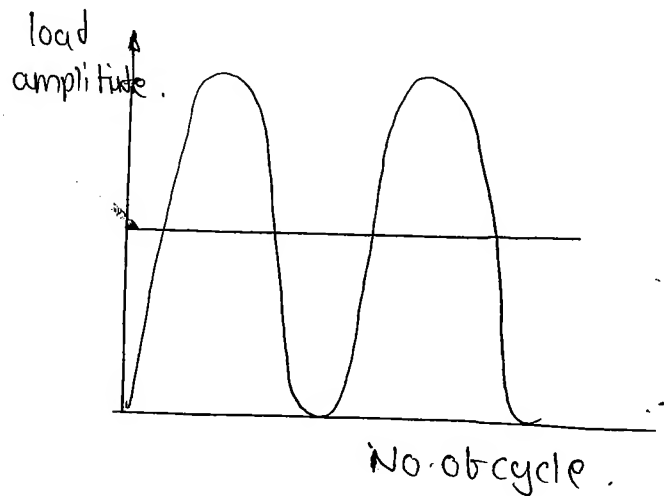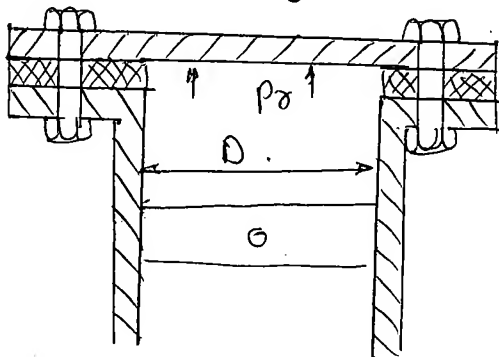
$$P = PA' \times \frac{\pi}{4} D^2$$

On the bolt the P is varying from min to maximum value.

Eccentric load on bolt Joint.

$$P = \frac{\pi}{4} d_c^2 \times n \times \frac{S_y t}{F_s}.$$

$$\tan\theta = \frac{a}{b}$$
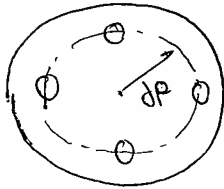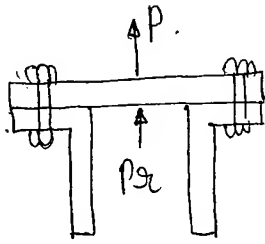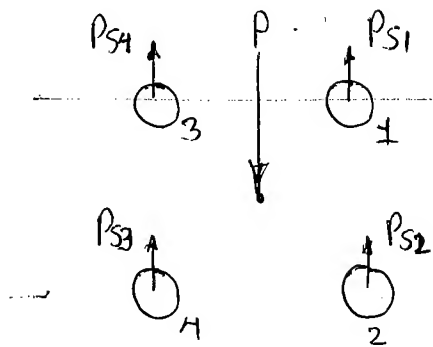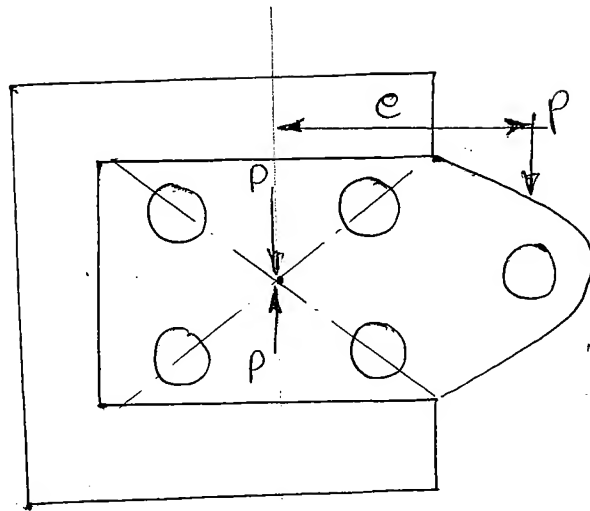
$$P_{S1} = \frac{P}{4}.$$

$$P_{S1} = P_{S2} = P_{S3} = P_{S4} = \frac{P}{4}.$$

Primary shear load
or
Direct shear load.
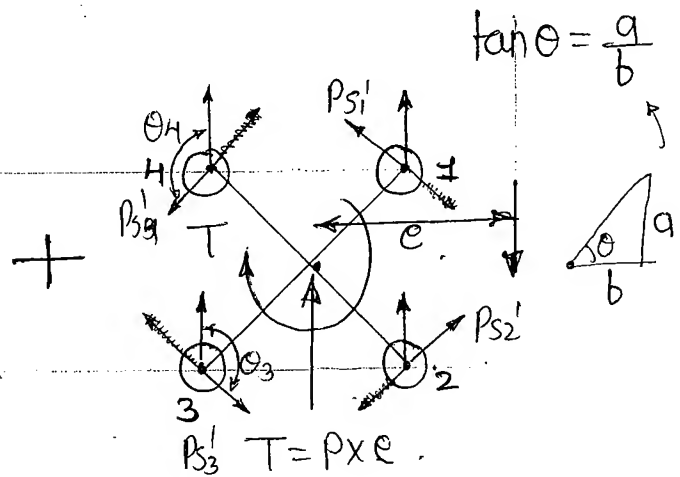
$$T = P \times e.$$

$$\frac{T}{J} = \frac{Z_2}{r}.$$

$$Z_2 \alpha r$$

Secondary shear load.

Resultant shear loads on the bolt

$$P_S = \sqrt{P_{S_1}^2 + P_{S_1}'^2 + 2 P_S \, P_{S_1}' \cos\theta}$$

$$P_S = \phi \, (P_S, P_S', \theta)$$

$$\theta \downarrow \quad \cos\theta \uparrow \quad P_S \uparrow$$

$$\boxed{(\theta_1 = \theta_2) < (\theta_3 = \theta_4)}$$

Q1. 1) The weakest bolt. — only one.

2) Total shear force on bolt (2) — $P/3$.

3) Least loaded bolt — 3

4) Primary shear load on bolt 3 — $P/3$

5] Secondary shear load on bolt ②. — 0.

1]
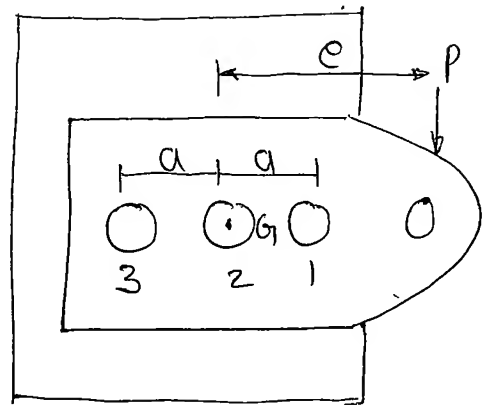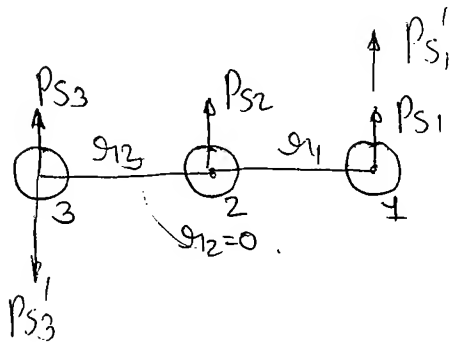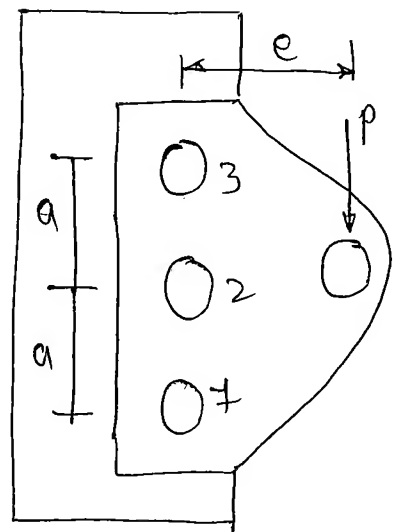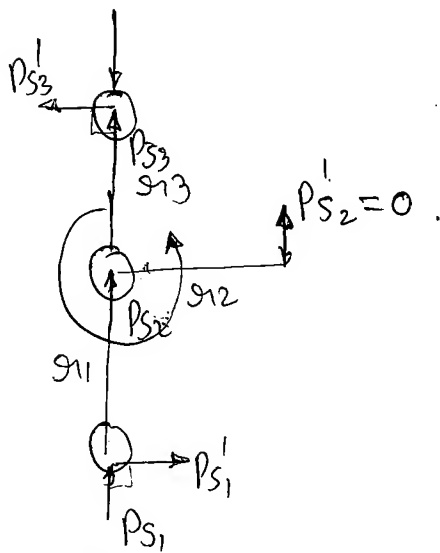
1]   1 and 3
2] $P/3$.
3]   2.
4] $P/3$
5]   0.
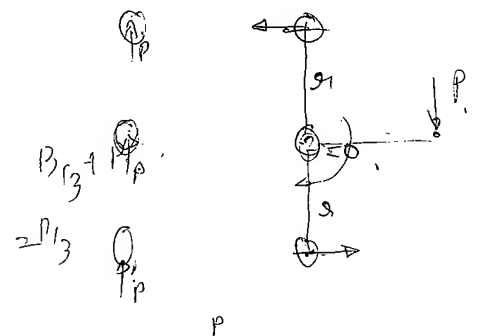
③ A bolt joint is shown in the fig
The maximum shear stress
in MPa in bolts A and B
respectively are.

3 holes of M10×1.75mm bolts

6mm thick

20 ↕
40 ↕
40 ↕
20 ↕

C
B x
A

F = 10KN.

150
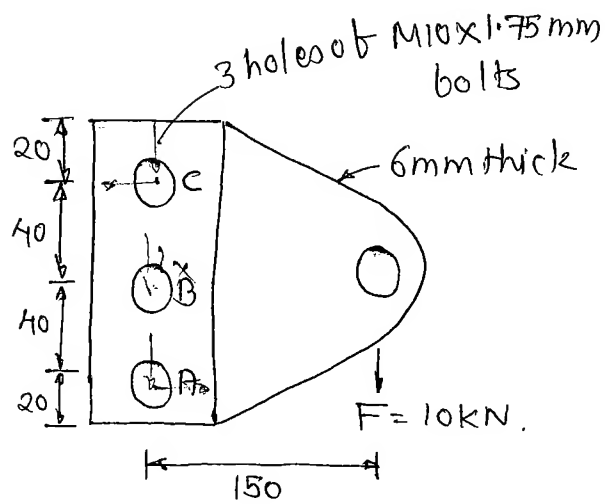
$$\tau_{total} = \frac{10}{\frac{\pi}{4}d^2} + \frac{16\,T}{\pi d^3}$$

$$= \frac{10\times10^3}{3\times\frac{\pi}{4}\times10^2} + \frac{16\times10\times10^3\times150}{\pi\times10^3}$$

$$= \cancel{127\cdot32RN} + 7639\cdot43$$

$$63\cdot66.$$

$$\tau_2 = 3333\cdot33 \text{ N/mm}^2.$$

$$\frac{T}{J} = \frac{f_s}{R}$$

$$T = \frac{\pi}{16}\,f_s\,d^3.$$

$$f_s = \frac{16\,T}{\pi d^3}.$$

$$= \sqrt{p^2+q^2+2pq\cos\theta}.$$

$$=$$

$$T = P_{s'_A}\times r_{A} + P_{s'_B}\, r_{B} + P_{s'_C}\, r_{C}$$

$$= K\, r_{A}\times r_{A} + K r_{B}\times r_{B} + K r_{C}\times r_{C}$$

$$P\times l = K\,(\,r_{A}^2 + r_{B}^2 + r_{C}^2\,)$$

$$K = \frac{P\times l}{r_{A}^2 + r_{B}^2 + r_{C}^2}$$

$$K = 0\cdot468.$$

$$P_{s'_A} = 0\cdot468\times40$$

$$= 18\cdot75 KN.$$

$$P_A = \frac{10}{3} = 3\cdot33$$

$$P_B = \frac{10}{3} = 3\cdot33$$

$$P_C = \frac{10}{3} = 3\cdot33$$

$$P = \sqrt{(P_{sA})^2 + (P_{BA'})^2}$$

$$= \sqrt{(3\cdot33)^2 + (18\cdot75)^2}$$

$$P = 19\cdot04\,KN,$$

$$\tau = \frac{19\cdot04}{\frac{\pi}{4}\times10^2}$$

$$\tau = 0\cdot242146\,KN$$

$$\boxed{\tau = 242\cdot46N.}$$

$$\tau_{SB} = \frac{P \tau_{SB}}{\pi/4 \, 10^2} \approx \frac{3.33 \times 10^3}{\frac{\pi}{4}(10)^2}$$

$$\tau_{SB} = 42.39 \text{ N/mm}^2$$

a] 242.6 , 42.5 b] 42.5 , 242.6 c] 42.5 , 42.5

d] 242.6 , 248.6 .

— x — x —

Bolts of Uniform strengths.

$U_{s} < U_{t}$

Shank   Thread.

$$U = \frac{\sigma^2}{2E}$$ d_h.

$$\sigma_t > \sigma_s .$$
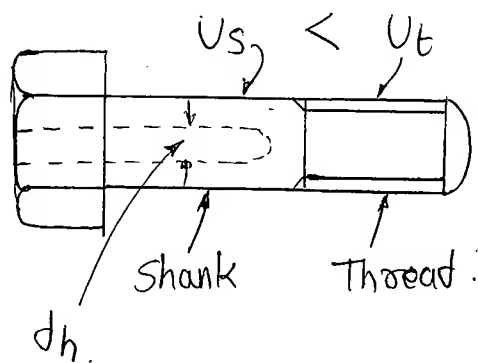
$$\sigma_t = \frac{P}{\frac{\pi}{4} d_c^2} \quad , \quad \sigma_s = \frac{P}{\frac{\pi}{4} d^2} .$$

$$\sigma_t \uparrow \quad \sigma_s \downarrow$$

If the energy stored in shank and bolt is same
the bolt is called bolt of uniform strength.

$$\boxed{U_s = U_t}$$

$$\frac{\pi}{4} d_h^2 = \frac{\pi}{4} d^2 - \frac{\pi}{4} d_c^2 .$$

$$d_h = \sqrt{d^2 - d_c^2}$$

d_r d_c
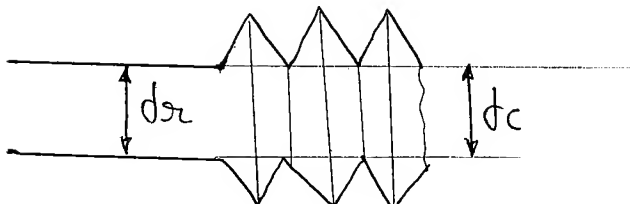
Total strain energy
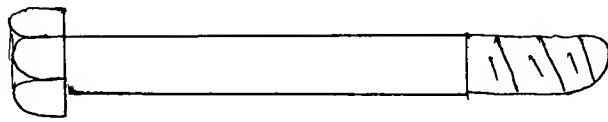
$$U = \frac{\sigma^2}{2E} \times A \times l.$$

DESIGN OF BEARINGS.

A bearing is a machine element which supports a moving moving member and confines its motion.
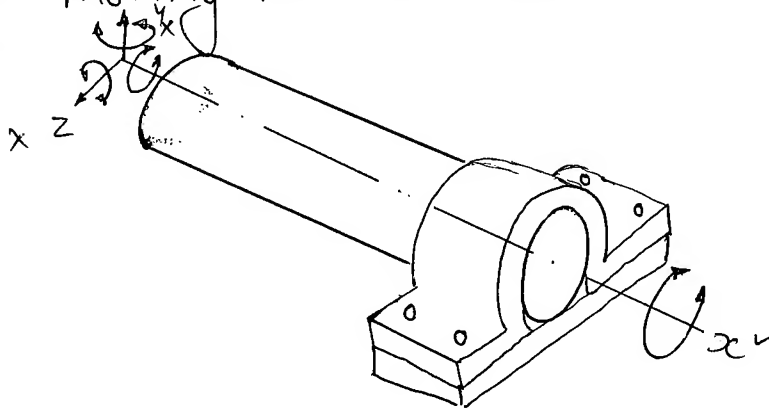
Bearings

1] Sliding contact bearings
   One surface slides over the other surface

2] Rolling contact Bearing
   Point or line contact.

Sliding Contact Bearing.

1] Journal Bearing

→ Radial loads
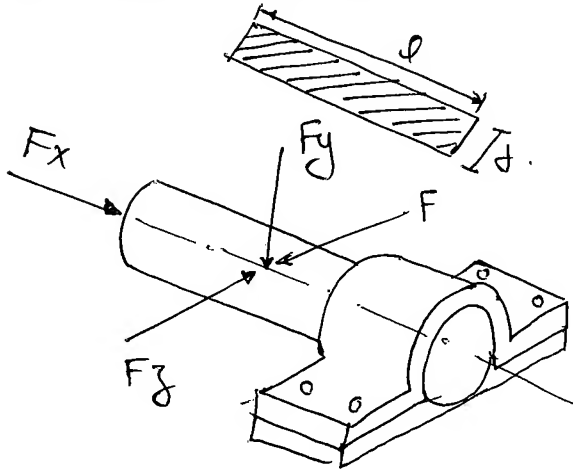
→ Fy, F, Fz - Radial loads

2] Collar bearings or Thrust Bearings

→ Axial loads.

→ Fx axial load

1] Journal Bearing
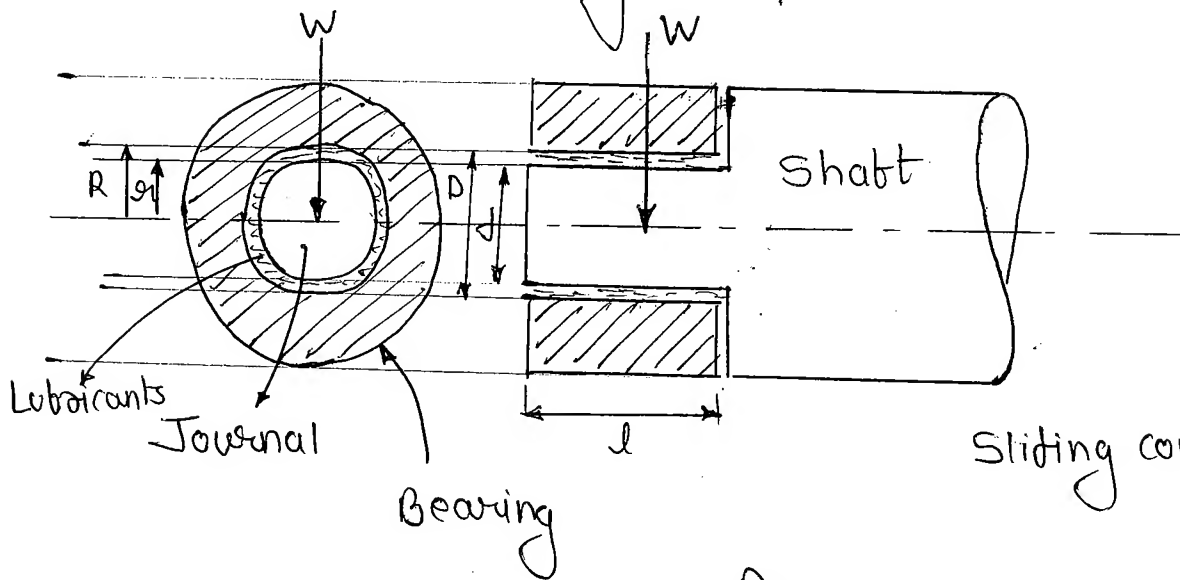
Sliding contact type.

d → dia of Journal
D → dia of Bearing.

Pressure

$$P = \frac{load}{Projected\ area}$$

$$= \frac{W}{ld}$$

Note :- load is always ⊥er to bearing

Clearance used

1] Radial clearance

$$c = R - r$$

2] Diametrical clearance

$$c_d = D - d$$
$$= 2c$$

Types of Journal Bearing.

① Full Journal Bearing.

→ Angle of contact 360°

→ Supports load in all radial directions
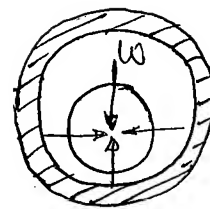
Ex:- IC engines, steam turbine, compressors etc.

Crank Shaft ← Journal bearings

← connecting rods
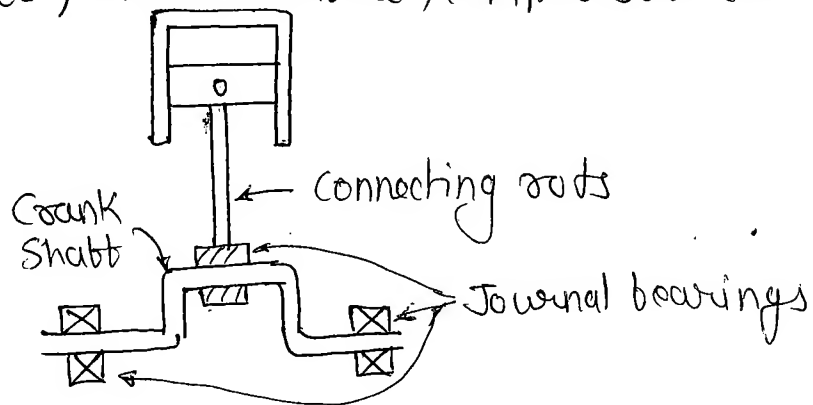
② Partial Journal Bearing.

→ Self cooled bearing.

→ load carry only one direction

→ β < 180°

Ex:- Rail road car axles.

Journal
Bearing
β = 120.

Bearing
casing
lubricants
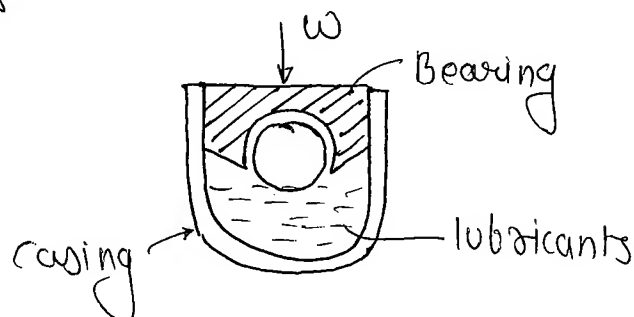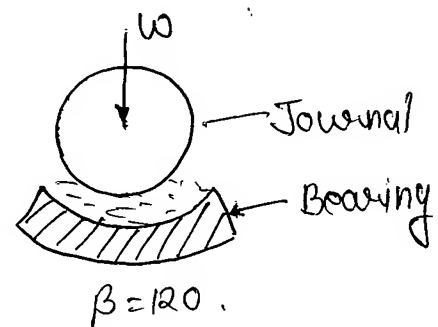
Fitted Journal bearing.

Radius of Journal is always equal to the radius (inner radius) of Bearing.

$$r = R.$$

Some type time eccentic alinement is taken into account.

$R > r$
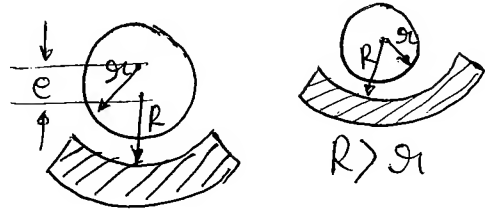
Methods of lubrications

a] Thick Film lubrication

→ Surface finish do not effect the performance hence intipentent of Surface Finish.

→ No contact.
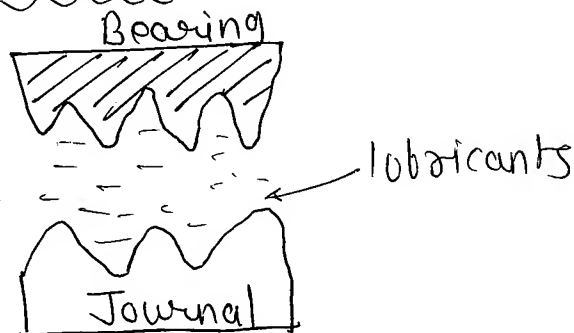
lubricants

b] Thin Film lubrication

→ Some contact is there
→ High surface finish
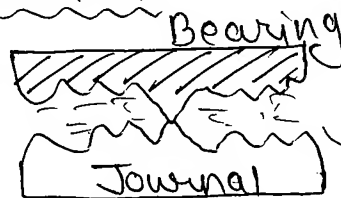
Thin film lubricants.

c] Solid lubrication

d] Elasto hydrodynamic & lubrication

used in point contact
type of location

$$P = \frac{W}{A}$$

$A \downarrow$ $P \uparrow$

Thick Film lubrication.

1] Hydrodynamic lubrication

2] Hydrostatic lubrication

Hydrodynamic lubrication

The load supporting fluid film is created by action of journal

→ No pump is used.

Hydrostatic Bearing

Load supported fluid film is generated by external pressure source.

→ Pump is used to generate the pressure.

Ex:- M/c bead

① working speed is low hence pressure is not generated by bearing

$$\frac{\partial P}{\partial x} = 0.$$

$$\frac{P_2 - P_1}{L}$$

Pressure gradient is zero in x direction

$$Z = \mu \frac{du}{dy}$$

μ = absolute viscosity

$N\text{-}s/m^2$
Pa-s or
$kg/m\text{-}s$
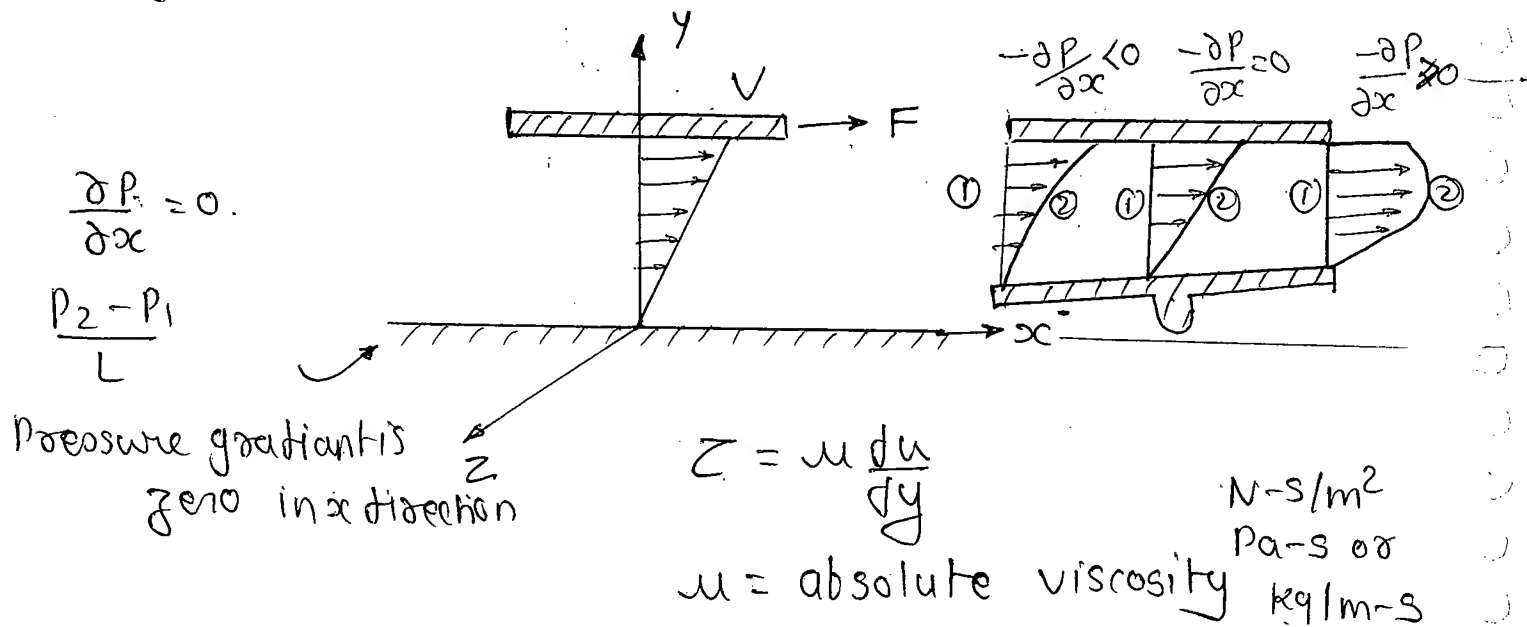
kinematic viscosity

$$\nu = \frac{\mu}{\rho} \quad m^2/s \quad \rho = \text{mass density}.$$

Because of plate velocity and adhesive force take fluid from low pr. to high pressure

$$-\frac{\partial P}{\partial x} < 0 \implies -\frac{\partial P}{\partial x} = 0 \implies -\frac{\partial P}{\partial x} > 0.$$

$$\frac{P_1 - P_2}{L} > 0.$$

→ Conditions for generating pressure

① Wedging action   Converging & diverging forces.

② Proper lubricant

③ Relative velocity.

Rest   Start   Running

Metal to Metal Contact

① Metal to Metal Contact

② Thin film lubrication.

$$V = \frac{\pi D N}{60}$$

N↑ V↑ P↑

③ load is balanced by

① No metal to metal contact

② Thick film lubration

③ load is balanced by pressure force
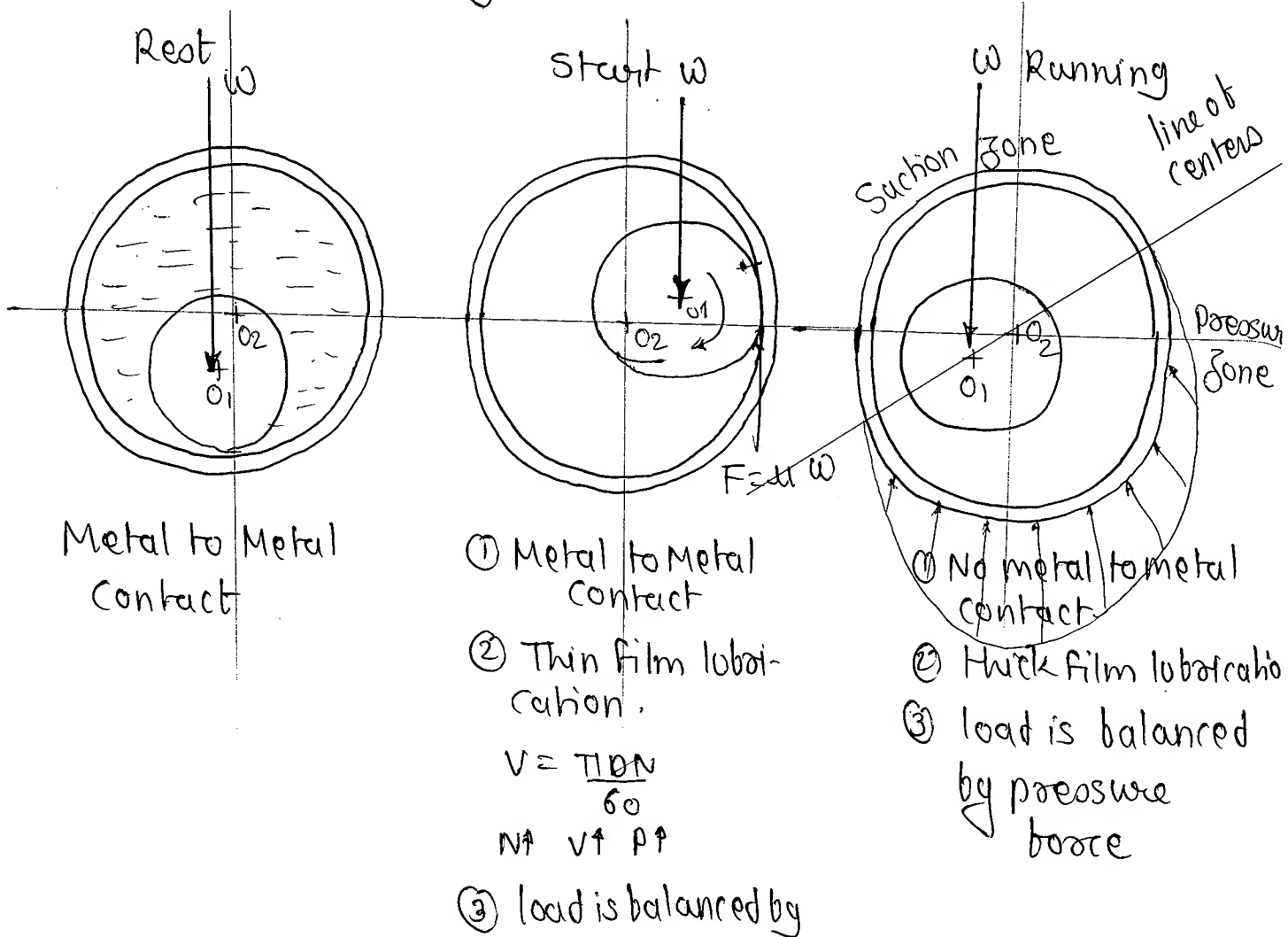
Performance of Journal Bearing.

→ Coeff of friction is main to determine the performance

Petroff's equation 1883

Torque $T = Force \times Radius$

$= Stress \times area \times$

$= Z \times 2\pi r L \times r$

$= \mu \dfrac{du}{dy} \times 2 \times \pi r \ell \times r$

$T = \mu \dfrac{2\pi r N_S}{C} \times 2\pi r \ell \times r$

$T_F = F \times W \times r$.

Pressure
$$P = \dfrac{W}{Projected \cdot area}$$

$$P = \dfrac{W}{2r \times \ell}$$

$W = P \times 2r\ell$

Friction torque

$T_F = F\omega \times r$

$T_F = f \times P \times 2r\ell \times r$

Performace of journal bearing.

$$F_P \times \cancel{2r} \cancel{l} = \mu \times \frac{2\pi \cancel{r} N_s}{c} \times 2\pi \cancel{r} \cancel{l} \times r$$

$$f = 2\pi^2 \left(\frac{\mu N_s}{P} \right) \left(\frac{r}{c} \right)$$

Assumption in this
    ① Lighter load is applied.
    ② Journal rotates concentrically $\Big\}$ Not Fit for Bearings

$\frac{\mu N_s}{P}$ ⟹ Bearing characteristic Number

$\frac{c}{r}$ ⟹ Radial clearance ratio.

2] Mckee's Equation.

→ At condition D.

Heat generated ↑
  temp ↑
  μ ↓
$\left(\dfrac{\mu NS}{P}\right)$↓

→ At condition D′

f↓ Hg↓ T↓ μ↑ $\left(\dfrac{\mu N3}{P}\right)$↑

→ At condition E

Hg↑ T↑ μ↓ $\left(\dfrac{\mu N3}{P}\right)$↓ f↑
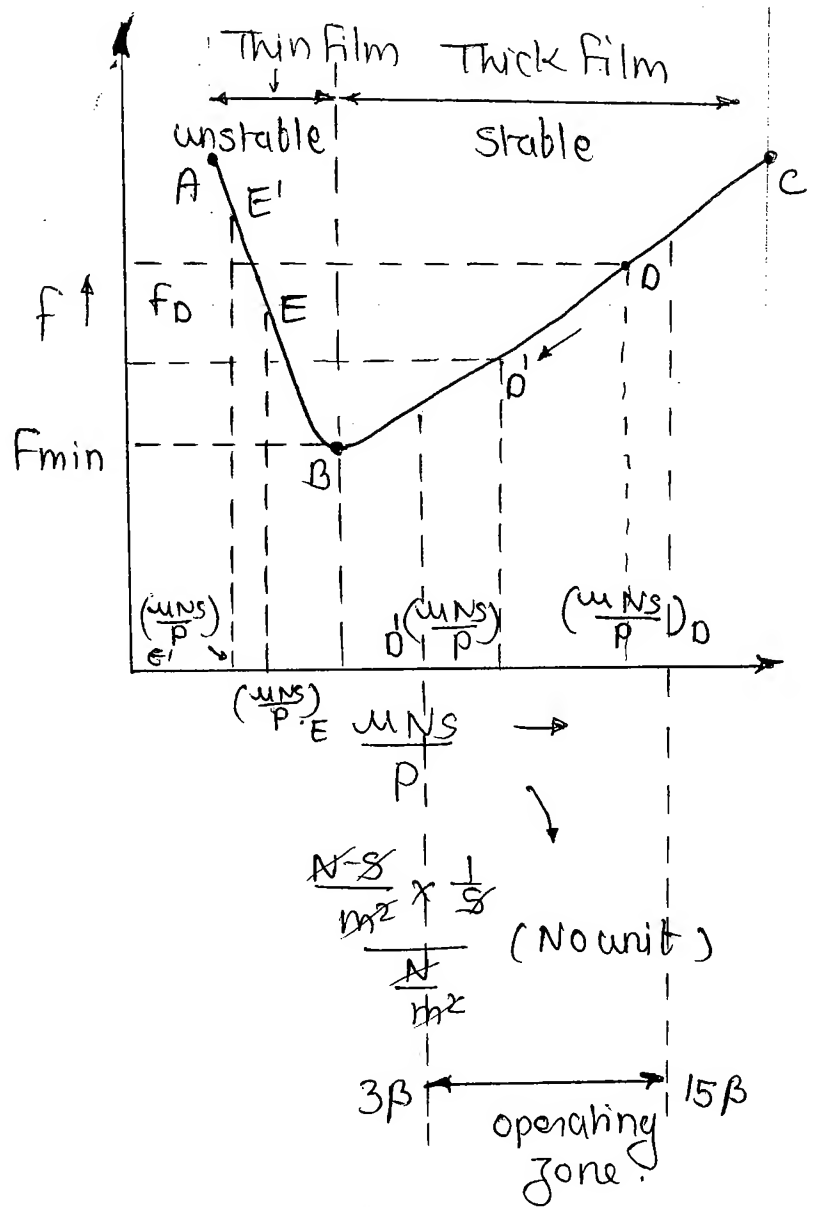
Compound Bearing

HSB + HDB
Hydrostatic + Hydrodynamic
  Bearing   Bearing.

$\dfrac{\mu NS}{P}$ corresponding to minimum cobt of

Foichion is called bearing modulus, β.

Bearing modulus only deal with Fmin

Operating zone

$$3\beta \lessgtr \left(\dfrac{\mu NS}{P}\right)_{operating} \lessgtr 15\beta$$

$$f = 0.326 \left(\frac{\mu N}{P}\right)\left(\frac{d}{Cd}\right) + K$$
$$= 0.33$$

N – revolution per min.

μ – $N\text{-}S/m^2$

N – rpm

P – N/m^2

K – Leakage factor.

$K = 0.002$

$$0.75 < \frac{l}{d} < 2.$$

Friction Torque

$$T = fW \times r$$

Power loss = Heat generated, H_g.

$$= \frac{2\pi N T}{60}$$
$$= fW \cdot v$$

$$v = \frac{2\pi d N}{60}.$$

H_d = Heat dissipated

$$= CA(t_b - t_a)$$

C = Heat dissipation coefficient

A = Projected area of beam

t_b = bearing temp.

t_a = Ambient air temp.

Pg no 54.

1] $d = 40mm$.
$W = 3KN$.
$P = 1.3 MPa$.

$$P = \frac{load}{Projected\ area}$$

$$1.3 = \frac{3 \times 10^3}{d \cdot L}$$

$$d \cdot L = \frac{3 \times 10^3}{2 \times 1.3}$$

$$L = \frac{3 \times 10^3}{40 \times 1.3}$$

$$L = 57.69.$$

$$\frac{L}{D} = \frac{57.69}{40} = 1.44. \qquad Ans\ b].$$

$\frac{L}{D} > 1$ long bearing
   Design bor less brichion under thin film lubrication
   Length is more, less leakage.

$\frac{L}{D} > 1$ short Bearing
   thick film lubrication
   more leakage

$\frac{L}{D} = 1$ Square bearing.

4]. $D = 25mm$

$L = 500.$ $\frac{L}{d} = 1.5$

$$P = \frac{load}{Projected\ area}.$$

2.2 KN, 300

A

B

500
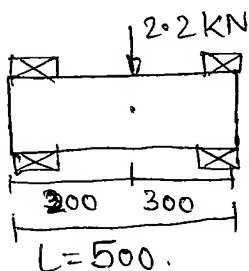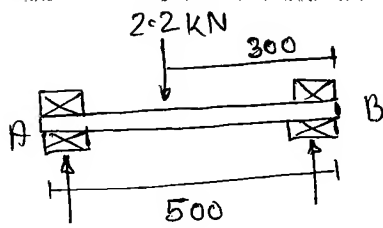
$$R_A + R_B = 2.2 \text{ KN}$$

$$R_A \times 500 + 2.2 \times 300 = 0$$

$$R_A = 1.32 \text{ KN}.$$

$$R_B = 0.88 \text{ KN}$$

$$P = \frac{load}{Ld} = \frac{1.32 \times 10^3}{1.5 \times 25 \times 25}$$

$$= 1.408.$$

$$\frac{L}{D} = 1.5$$

$$L = 1.5D$$

⑤ $d = 75mm$

$N_1 = 300 \text{ rpm}$

$\mu_1 = 0.06$ pa-sec

$C_1 = 0.2mm$

$P_1 = 1.4 \text{ MPa}.$

$N_2 = 400 \text{ rpm}$

Bearing contact number

$$= \frac{\mu_1 N_1}{P_1}$$

$$= \frac{0.06 \times 300}{1.4}$$

$$= 12.85.$$

To stable the bearing

$$\frac{\mu_1 N_1}{P_1} = \frac{\mu_2 N_2}{P_2}.$$

$$12.85 = \frac{0.06 \times 400}{P_2}$$

$$\boxed{P_2 = 186. MPa}$$

⑦. $d = 150mm$

$l = 225mm$

$W = 9KN$

$N = 1000 \text{ rp m}$

$C = 0.075mm$

$h_d = 90 \text{ KJ/min}$

$\mu = ?$

$$h_d = CA(T_b - T_a)$$

$$h_g = f\omega V. \quad J/s. \text{ or } J/min$$

$$h_g = h_d.$$

$$f \times 9 KN \times \pi \times 150 \times 10^{-3} \times 1000 = 90 \text{ KJ/min}$$

$$f = 0.02$$

$$f = 0.326 \left(\frac{\mu N}{P}\right)\left(\frac{d}{G}\right) + K$$

$$(0.02)\ f = 0.032 \times \left(\frac{\mu \times 1006}{\underset{\underset{m}{0.150 \times 0.22}}{9 \times 10^3\ N}}\right) \times \frac{150}{2 \times 0.075} + 0.002$$

$$\mu = \cancel{0.0035}\ Pa\text{-}s$$
$$0.0157$$

⑨

$d = 50mm$

$l = 75mm$

$P = 2\ MPa.$

$N = 500 rpm$

$C = 11.6\ w/m^2\text{-}k\overset{\circ}{c}$

$A = 8 \times l d.$

$ta = 28°c$

$f = 0.0015$

$$Hd = c\hat{A}(tb - ta)$$

$$Hg = Fw \times V$$
$$= 0.0015 \times P \times \cancel{A \theta l d} \times \frac{\pi d N}{}$$
$$= 0.0015 \times \underset{2}{2} \times 75 \times 50 \times \pi \times 50 \times \underset{60}{\overset{\times 10^{-3}}{500}}$$
$$= 883.57 \times 10^3$$

$$Hd = Hg$$
$$\frac{883.57 \times 10^3}{1000} = 11.6 \times 8 \times (75 \times 10^{-3} \times 50 \times 10^{-3}) \times (tb - 28)$$

$$Hg = 0.0015 \times 2\ \frac{N}{mm^2} \times \frac{50 \times 75}{mm^2} \times \pi \times 50 \times 10^{-3} \times \left(\frac{500}{60}\right)$$
$$= 14.72.$$

$$Hg = Hd.$$
$$14.72 = CA\ (tb - ta)$$
$$14.72 = 11.6 \times 8 \times 50 \times 10^{-3} \times 75 \times 10^{-3}\ (tb - 28)$$
$$tb = 70.31°c$$
$$\boxed{tb = 343\ .311\text{k}}$$

Sommerfield Number / Bearing number.

$$S = \left(\frac{\mu N_S}{P}\right)\left(\frac{r^2}{C}\right).$$

$\hookrightarrow$ Radial clearance

Pg 60

Pno 55]   W = 50 KN,

L = 150 mm ⎱ squre
d = 150 mm ⎰

C = 0.15 mm
bmin = 0.03 mm
μ = 8 Cp. = 8×10⁻³
S = 0.0446
N = ?

$P = \dfrac{\text{load}}{\text{Projected area}}$

$= \dfrac{50\times10^3}{150\times10^{-3}\times 150\times10^{-3}}$

$= 6666.6\times10^3$

$$S = \left(\frac{\mu N_S}{P}\right)\left(\frac{r^2}{C}\right)$$

$$0.0446 = \left(\frac{8\times10^{-3}\times N_S}{6666.6\times10^3}\right)\times\left(\frac{150}{2\times0.15}\right)^2$$

~~0.0446~~ $= 3\times10^{-4}\,N_S.$

$N_S = 49.55$

$N = N_S\times60$

$N = 2973.33\,rpm$

Altitute or Eccentricity ratio ($\frac{\xi}{\xi}$, ξ)

ho - min oil film thickness

$$\xi = \frac{\text{Eccentricity}}{\text{Radial clearance}} = \frac{e}{c}$$

$$= \frac{c - ho}{c}$$

$$= 1 - \frac{ho}{c}$$

$R = e + r + ho$

$R - r = e + ho$

$c = e + ho$

If $e = 0$, $c = ho$

(Not possible)

ϕ = attitude angle.

- If the ϕ maintain constant then that type of bearing is stable bearing.

- If the ϕ not maintain constant oil whirl can balance fluid forces created due to vibration.

Line of centers
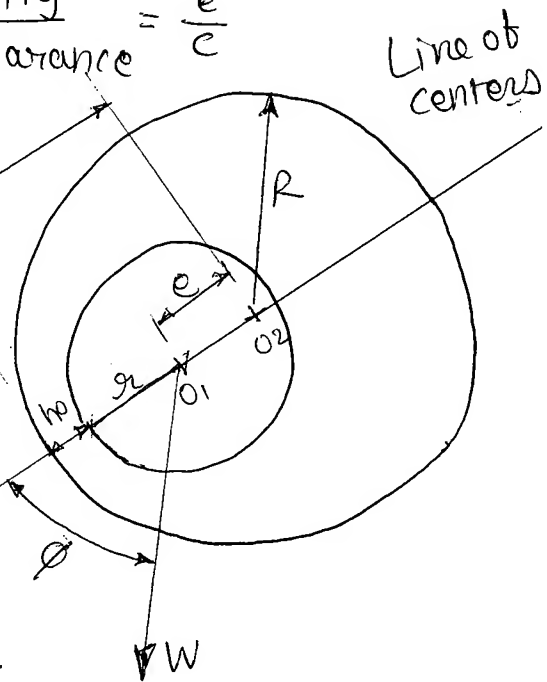
ϕ = constant
Stable bearing

Oil whirl

unbalanced fluid forces

Oil whip.

Fluid forces frequency matches with frequency bearing material.

6]

$$\varepsilon = \frac{\dfrac{h_0}{c}}{} = 0.8$$

$$\varepsilon = 1 - \frac{h_0}{c}$$

$$0.8 = 1 - \frac{h_0}{c}$$

$$\frac{h_0}{c} = 0.2$$

Rolling Contact Bearings.

Rolling contact Bearings (Antifriction bearings)

① **Ball Bearing**

→ Point Contact

② **Roller Bearings**

→ line contact

More load carring capacity than roller Bearings

outer ring

B

D

d

Inner ring

outer race .

Inner race

rib

R

Race

R > R rib .
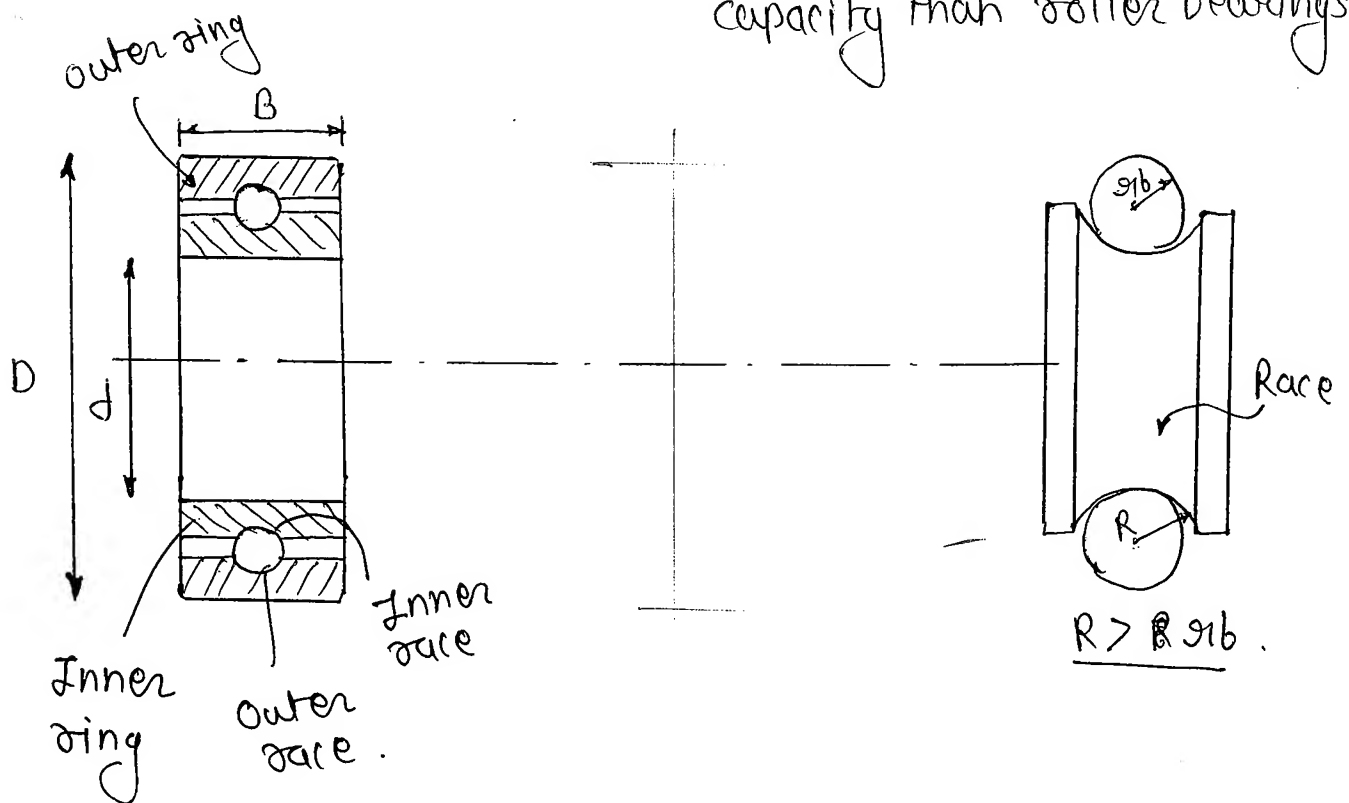
d = Inner dia or dia of shaft

D = outer dia

B = axial width .

High chromium steel – outer ring

More rigidity

Snap ring.

Interference fit.
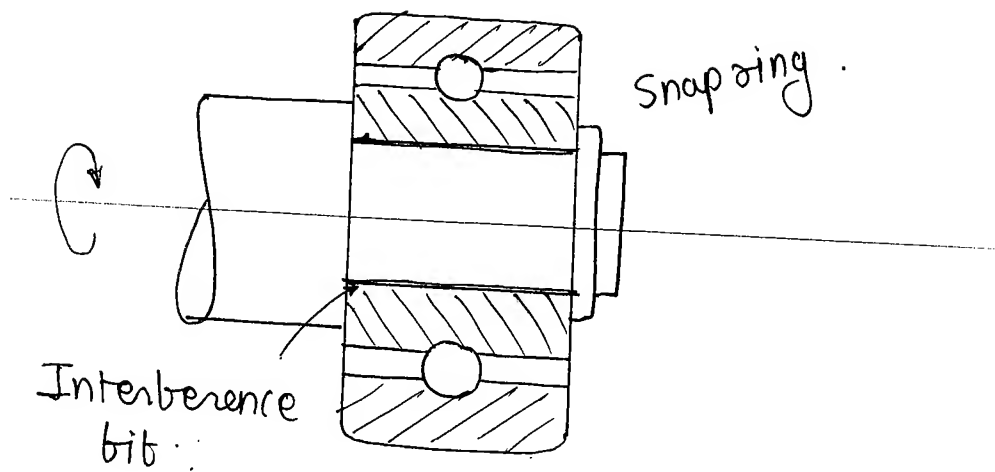
Types of ball bearings.
1. Deep groove ball bearings
2. Angular contact ball bearings
3. Self aligning " "
4. Thrust ball bearings.

Deep groove
¼ deep both side

Radial & axial load

axial load ↔ ↑ Radial load

Angular contact B.

In this case site is use to insert balls

Design to carry more axil & Raial load.

$\dfrac{Fa}{Fr} > 1$

↑ Radial load
axial load →

hance two bearing is used in pairs
Side by side

Self align

least cobf of friction

Radial load and axial load

It will work in angular missalinement

angular missalingment

Thrust ball bearing

only support axial load

axial load in both direction ↔

Roller contact bearing

1] Cylindrical Roller bearing
2] Tapered Roller bearing
3] Spherical Roller bearing
4] Needle roller bearing

Cylindrical Roller	Tapered Roller	Spherical Roller	Needle roller

$\frac{l}{d} \geqslant 4$
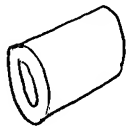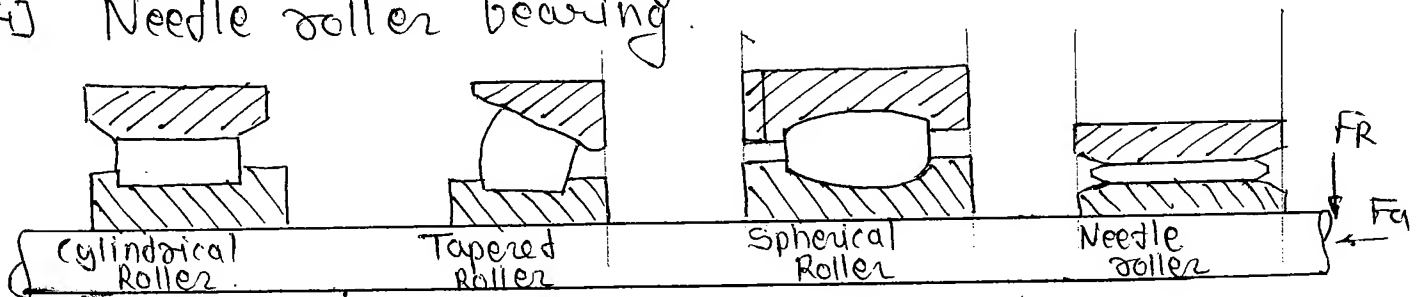
Only medium to heavy radial load

Both radial and axial load

Ex- turbine, automobile axle

Magnitude is heavy.
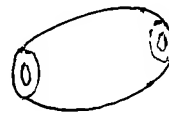
Radial and axial loads

Angular missalignment upto $\alpha = 3°$ it sustain
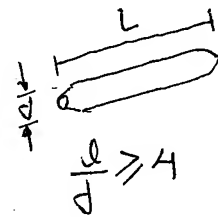
less radial space but more axial space is required

Perfect alignment is req. otherwise it is bent.

Heavy radial load and No axial load can sustain

occillating load can be sustain by this

Ex:- Hevy duty diesel engine

Piston pin bearing can be replaised by this
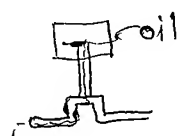
Rolling Contact bearing.

Designation.

1) SKF -

X X X X

Type of Bearing   Load carrying Capacity   X5   Inner dia or Shaft dia.

100   200   300   400
Extra   Light   Medium   heavy
light   ~~light~~
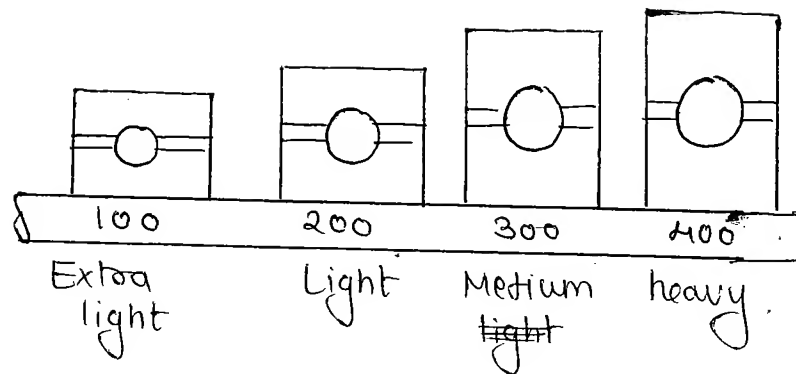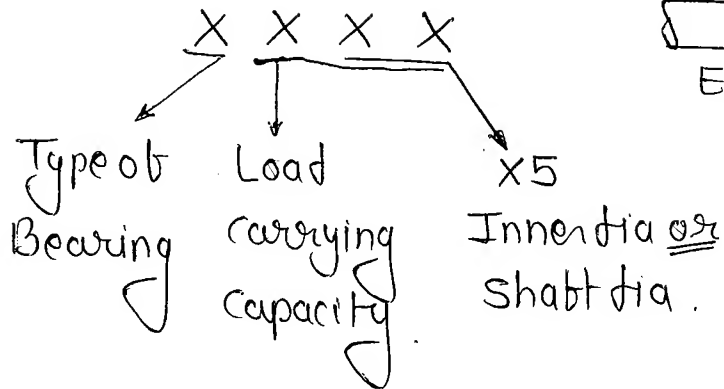
Ex:- 6307

7×5 = 35 mm dia

Medium duty

2) BIS - Antifriction Bearing manufacturing association (AFBMA)

40 BC X X

Ball Bearing

Deep groove

Width Series

diameter Series.
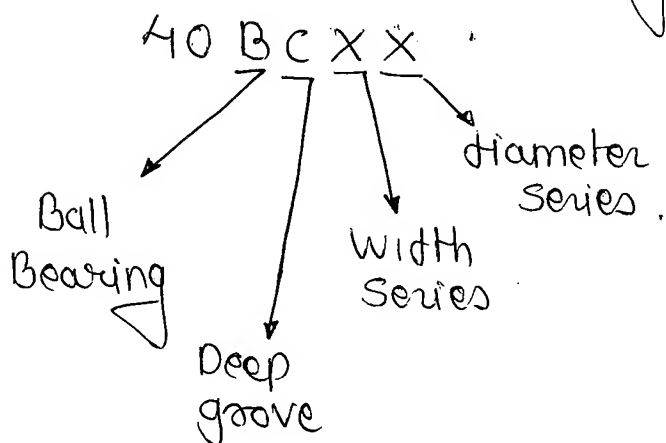
Ex:- ① 6307   07×5

BIS   35 BC 03

② 6410

BIS   50 BC 04

3) SKF
   6 112
   6 212
   6 312
   6 412

BIS   100 series   SKF

12 x 5 BC 01 = 60 BC 01 - light
             = 60 BC 02 - light
                60 BC 03 - Medium
                60 BC 04 - heavy

Designation   d   D   B   r   Co   C   N

- d → Inner dia of Bearing
- D → Outer dia of Bearing
- B → Axil width
- r → Fillet radius
- Co → Static load carrying Capacity
- C → Rated load Dynamic load carrying Capacity
- N → Maximum speed
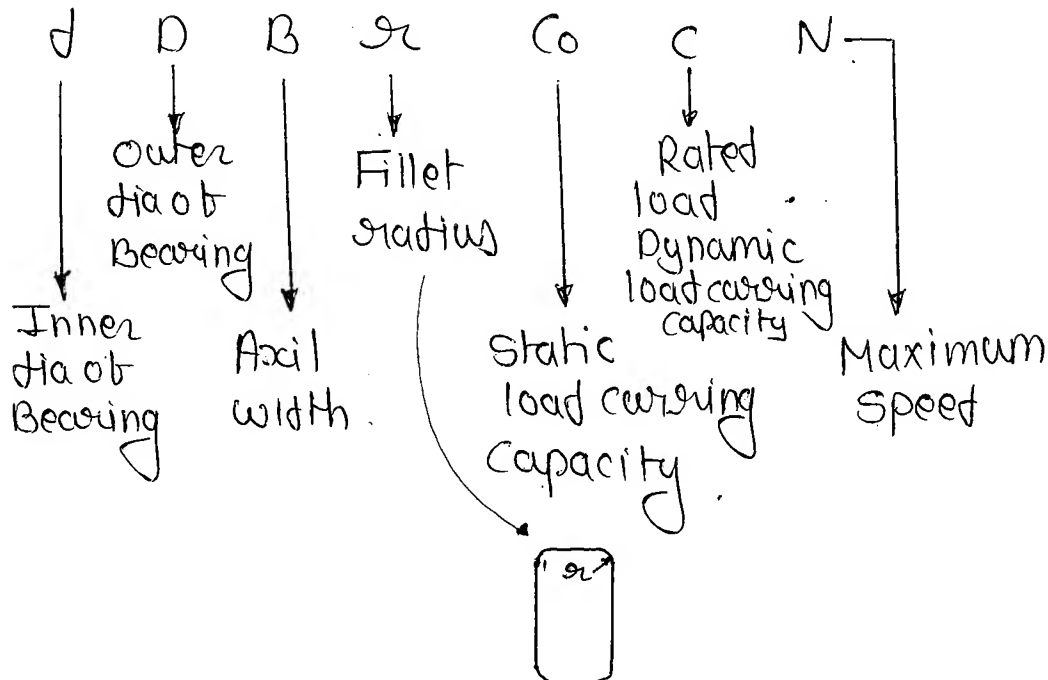
Static load carrying capacity [Co]

Load required to produce permanent deformation in the balls and races at the most heavy stress point of contact equals to 0.0001 D, where D is the dia of ball in mm.

Brinelling failure
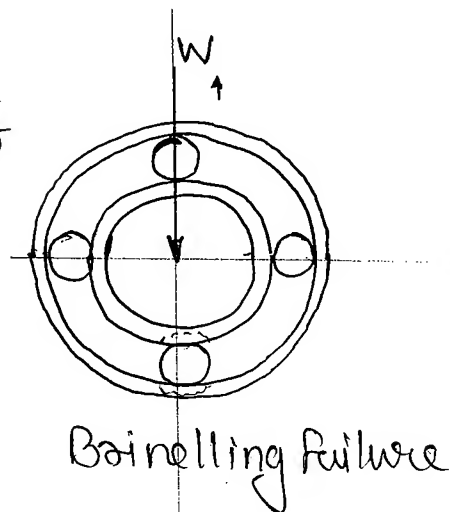
$$Co = \frac{K D^2 Z}{5} \text{ — For ball bearing}$$

K = Constant depends on ball dia.
D = Dia of ball
Z = No of balls.

$$Co = \frac{K D Z L}{5} \text{ — For roller bearing}$$

L = length of roller.

Rated Life.

Life → No. of revolution completed before the evidence of Fatigue crack.

Rated life

L_{10} → 90% are in good condition

L_{50} → 50% are in good condition.

↓ avg life.

$$L_{50} = 5 \times L_{10}$$

min load

max load

hence Fatigue Failure

w

| Motor | 1 | 2 | 10 | · · · · | 100 |
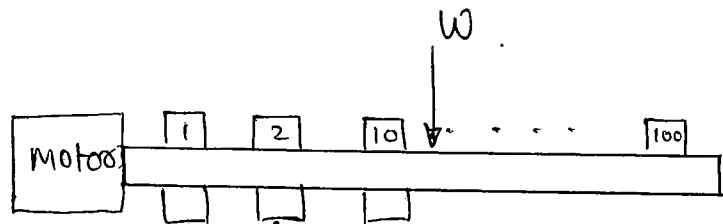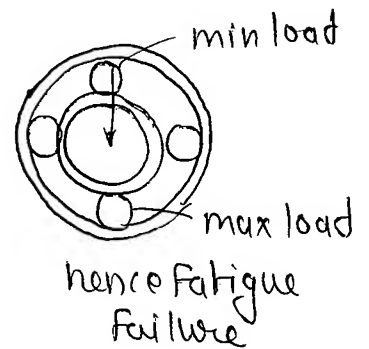
vibration sensing device

1 bearing is fail then 2 · · · · · up to 10 bearing is fail that is called L_{10} life.

The load is applied and calculated the rev. up to 10th bearing fail. and the the load value is reduce to cover 10^6 rev. and corrosponding load is calculated

Rating Life (L_{10})

This is the life of group of apparently identical Ball or roller bearings, in number of revolution or hours at a given speed that 90% of bearings will complete or exceed befor any indication of Failure

Basic load Rating (c)

It is that load which a group of apparently identical bearings can withstand for a rating life of one million revolutions.

$$L = \left(\frac{C}{P}\right)^K$$

L = Rated life   K = 3 For Ball bearing

C = Basic load Rating   K = $10/3$ For Roller Bearing

P = equivalent load.

Equivalent load.

$$P = (X V F_r + Y F_a) C_s.$$

X = radial load factor.

V = race Rotation factor

    = 1 For inner race rotation

    = 1.2 For outer race rotation

F_r = radial load.

Y = Axial load factor

F_a = Axial load.

C_s = Service factor

Life in million Revolution

$$L = \frac{60 \, n \, L_H}{10^6}.$$

n = Speed rpm

L_H = Life in hours.

9]

$$\frac{6205}{200 \text{ series}} \rightarrow 10.8 \text{ KN}.$$

$$\frac{6305}{300 \text{ series}} \rightarrow 16.2 \text{ KN (rest are less then } 10.8 \text{ KN)}$$

Ans d]

10]. 6210 bearing.

3 ←

C = 22.5 KN

L = 27 million rev

P = ?

$$L = \left(\frac{C}{P}\right)^K$$

$$27 = \left(\frac{22.5}{P}\right)^3$$

$$P = 7.5 \text{ KN}.$$

$$L_{50} = 5 \times L_{10} \quad \text{— Rated life.}$$

$$L = k\left(\frac{c}{p}\right)^k$$

$$P = X \, V \, F_{9t} + Y \, F_a.$$

$$L = \frac{60 \, n \, L_H}{10^6}.$$

Pg no 67.

13] $F_{9t} = 2.5 \, KN.$

$F_a = F_t = 1.5 \, KN.$

$C_s = 1.5$.

40 hrs/week for 5 years

$n = 1000 \, rpm$

$X = 0.56$

$Y = 1.4$

$$t = \left(\frac{c}{p}\right)^k$$

$$= \left(\frac{1.5}{2.5}\right)^3$$

$$=$$

$$L = \frac{60 n \times L_H}{10^6} = 60 \times 1000 \times$$

$$n_H = 40 \times \frac{hr}{week} \times 52 \, week \times 5 \, years$$

$$= 10400 \, hr$$

$$L = \frac{60 \times 1000 \times 10400}{10^6}$$

$$L = 624$$

$$L = \left(\frac{c}{p}\right)^k$$

$$P = (XV \, F_{9t} + Y \, F_a) \, C_s.$$

$$= (0.56 \times 1 \times 2.5 + 1.5 \times 1.4) \, 1.5$$

$$= 5.25 \, KN.$$

$$624 = \left(\frac{c}{5.25}\right)^3$$

$$C = 44.86$$

(12)

$C = 48.545 \text{ kN}$.

$LH = 6000 \text{ hrs (Avglibe)}$. $P = (X \vee F_{91} + Y F_a) G$

$n = 500 \text{rpm}$.

$$L = \frac{60 \times 500 \times 6000}{10^6}$$

$$L = 180 \text{ million rev}.$$

$$L = \left(\frac{C}{P}\right)^k$$

$$\frac{180}{5} = \left(\frac{48.545}{P}\right)^3 = L_{10}.$$

$$P = 14.7 \text{ kN}.$$

Beaving Subjected to a cyclic loads.

Assume a bearing is subjected to load.

P_1 at n_1 rpm   For α_1 % of time.

P_2 at n_2 rpm   For α_2 % of time.

.
:

P_x at n_x rpm   For α_x % of time.

It the bearing is subjected to only load P_1, then its libe is, $L = \left(\frac{C}{P_1}\right)^3 \times 10^6$ revolution

Libe consumed during one revolution, $\dfrac{1}{L_1} = \dfrac{P_1^3}{C^3 \times 10^6}$.

A1 load

P_1 . speed n_1, rpm = α % of time

2000   7 min.

total revolution $N_1 = \alpha_1 n_1$ rev.

N_1 revolution, $\dfrac{N_1}{L_1} = \dfrac{N_1 P_1^3}{C^3 10^6}$.

Life consume for N_2 revolution $= \dfrac{N_2}{L_2} = \dfrac{N_2 P_2^3}{C^3 10^6}$.

$$\dfrac{N_1 P_1^3}{C^3 10^6} + \dfrac{N_2 P_2^3}{C^3 10^6} + \cdots + \dfrac{N_x P_x^3}{C^3 10^6} = \dfrac{N P_e^3}{C^3 10^6}.$$

$$N = N_1 + N_2 + N_3 + \cdots + N_x$$

$$P_e \text{ (equivalent load)} = \left[\dfrac{N_1 P_1^3 + N_2 P_2^3 + \cdots + N_x P_x^3}{N_1 + N_2 + \cdots + N_x}\right]^{1/3}$$

P_e = Equivalent load or Cubic mean load.

Pg no. 67.
Q 19 and 20.

Element no.	% of element time	load (KN)	Speed (rpm)
1	30	5	900
2	40	7	1440
3	30	3	720.

$N_1 = \alpha_1 n_1 = 30 \times 900$

$N_2 = \alpha_2 n_2 = 1440 \times 40$

$N_3 = 720 \times 30$.

$$P_e = \left[\dfrac{\dfrac{30 \times 900}{100} \times (5)^3 + \dfrac{40 \times 1440}{100} \times (7)^3 + \dfrac{30 \times 720}{100} \times (3)^3}{\dfrac{30 \times 900}{100} + \dfrac{40 \times 1440}{100} + \dfrac{720 \times 30}{100}}\right]$$

$$L_{10} = \left(\frac{C}{Pe}\right)^3 = \left(\frac{16.6}{6.066}\right)^3$$

$$= 20.46 \text{ million rev}$$

Compression

Bearing

Journal

W

Pump

Hydrostatic Bearing

load carrying Capacity

RCB

HDB

HSB

Speed (N)
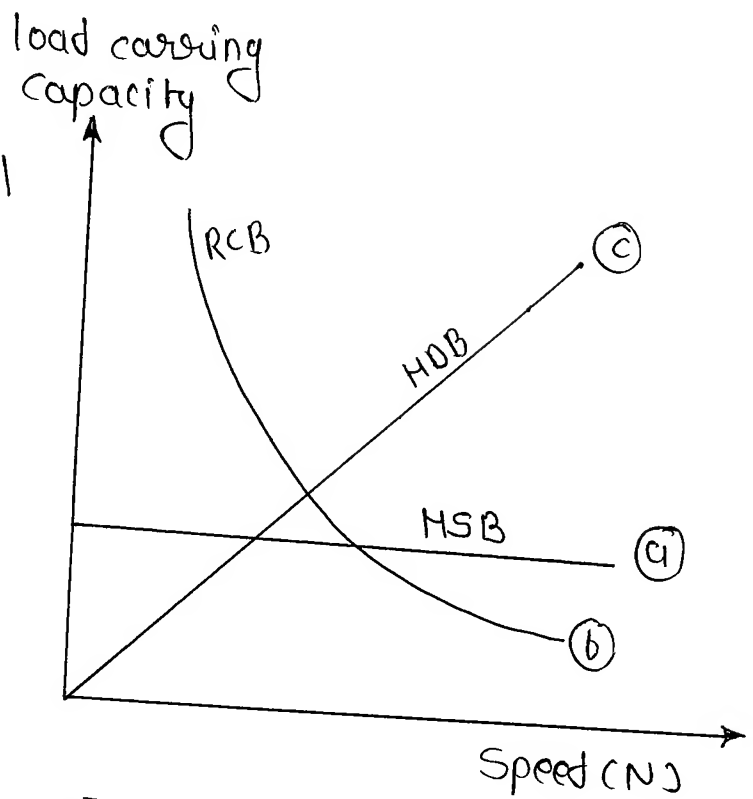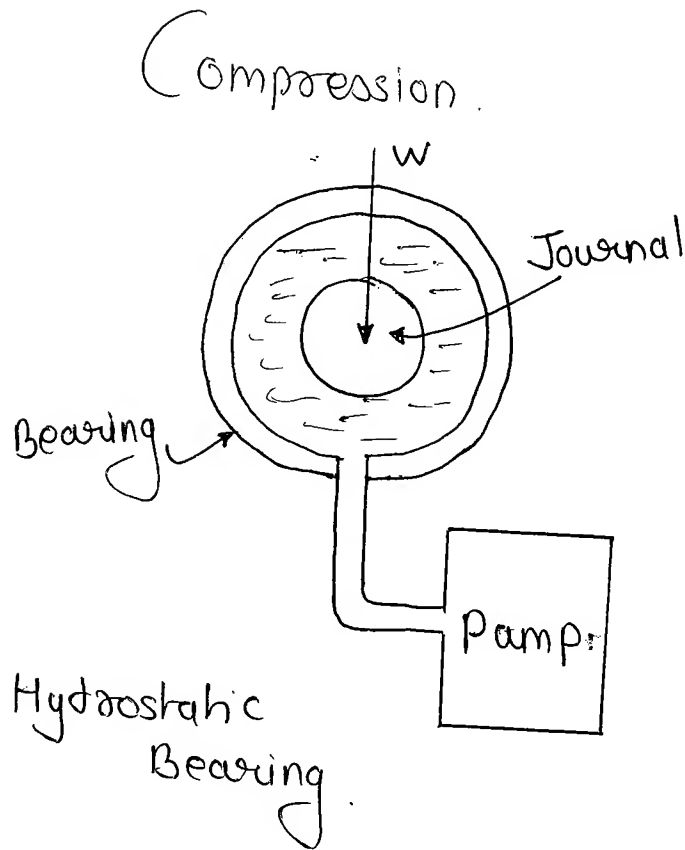
a) Hydrostic
b) Rolling contact
c) Hydrodynamic.

f ↑
Coff of friction.

Speed
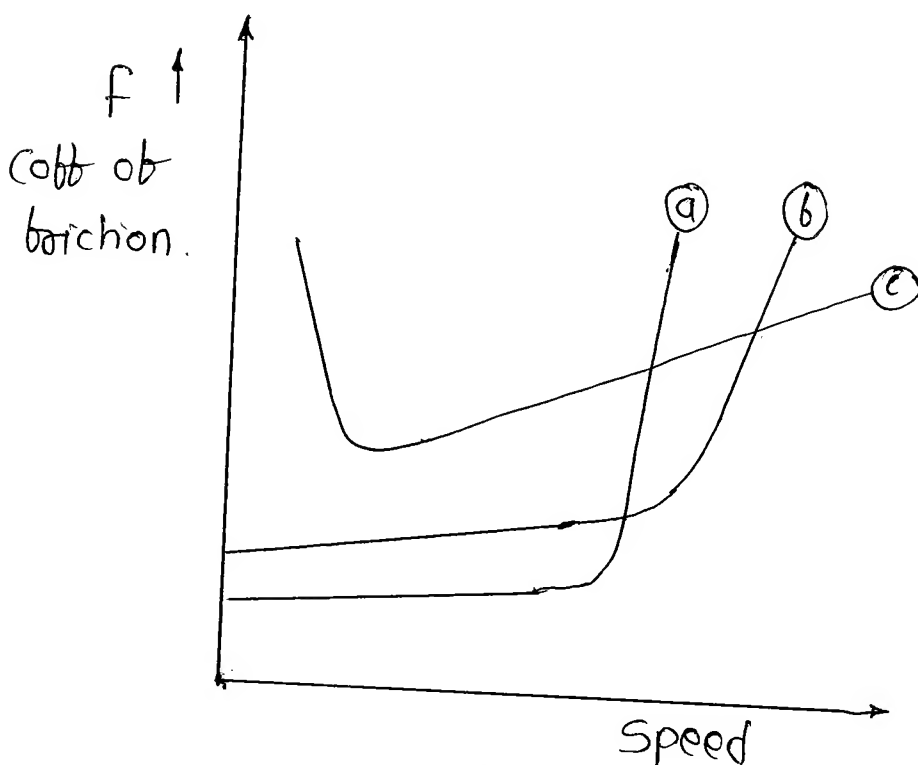
a – Ball
b – Roller
c – Hydrodynamic

3] Starting Torque.

Hydrodynamic bearing (HDB)
　　　　- High - because of high starting friction.

Rolling contact bearing (RCB)
　　　　- low - because of low starting friction.

4] Space.

→ Hydrodynamic B
　　　　More axial space, less radial

→ Rolling. C.B
　　　　More radial and less axial.

DESIGN OF BRAKES.

A brake is a device used to control the speed of a moving member by:
1] Providing suitable decceleration
2] Absorbing Energy.

Energies.

1] Kinatic energy of translation, $K \cdot E_T = \frac{1}{2} m (v_1^2 - v_2^2)$
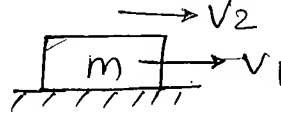
2] K·E of Rotation, $KE_R = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$
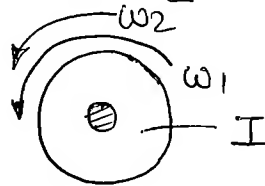
3] Potential Energy, $PE = wh$
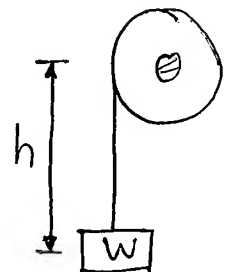
Classifications of Mechanical Brakes.

Brakes.

Radial Force is applied   Radial → Axial.   Axial Force is applied

External   Internal
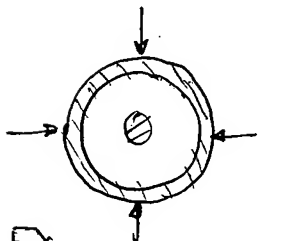
Ex
Shoe brake
band brake
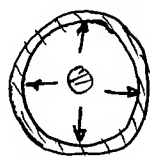
Ex:- Internally expanding Shoe brake

Ex:- Disc Brake

Friction pad

Friction pad

steel disc
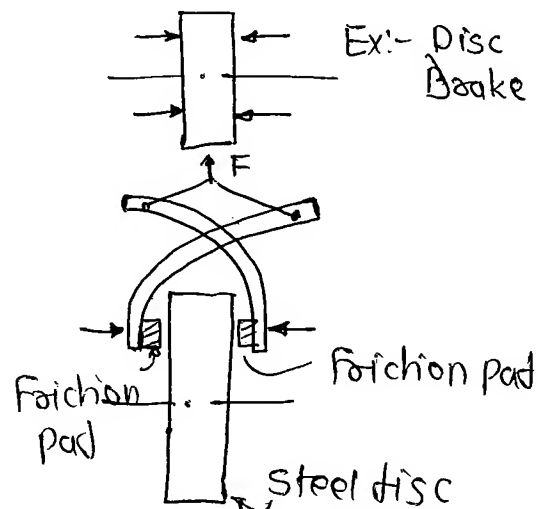
Principle of Shoe Brake.
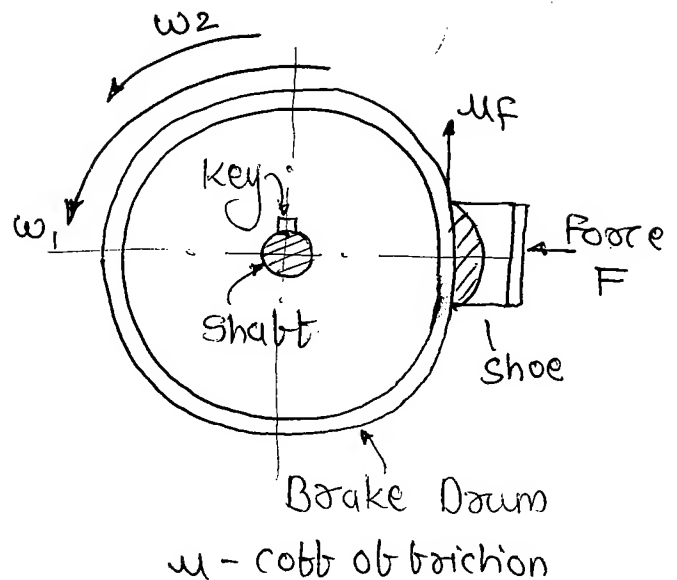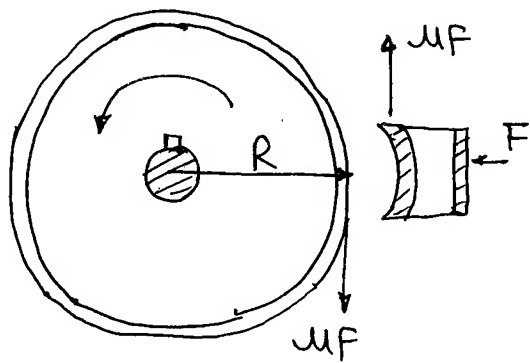
→ Friction Torque.

$$T_B = \mu F \times R.$$

→ Energy dissipated.

$$\Delta E = T_B \times \theta.$$

① $\quad v = u + at$

$$w_2 = w_1 + \alpha t.$$

$$\alpha = \frac{w_2 - w_1}{t}$$

② $\quad v^2 = u^2 = 2as$

$$w_2^2 - w_1^2 = 2\alpha \theta.$$

$$w_2^2 - w_1^2 = 2 \times \frac{w_2 - w_1}{t} \times \theta$$

$$\theta = \left(\frac{w_1 + w_2}{2}\right) t.$$

→ Heat generated.

$$Hg = T_B \times \left(\frac{w_1 + w_2}{2}\right) t$$

$$= \mu F \times R w \times t$$

$$= \mu F V \times t.$$

$$Hg/sec = \mu F V$$

$$w = \frac{w_1 + w_2}{2}$$

$$V = R w$$

$$p - \text{Pressure.}$$

Shoe or Block Brake

Case I.

$$\varepsilon M_{pivot} = 0.$$

$$P \times l - R_N \times x = 0$$

$$P = \frac{R_N x}{l}.$$

Pivot

l

x

$P = \text{effort}$

μR_N
shoe

Lever

2θ

R_N

Brake drum

Case II

$$\varepsilon M_{pivot} = 0.$$

$$\underbrace{P \times l}_{AM} - \underbrace{R_N x}_{} + \underbrace{\mu R_N \times a}_{FM} = 0.$$
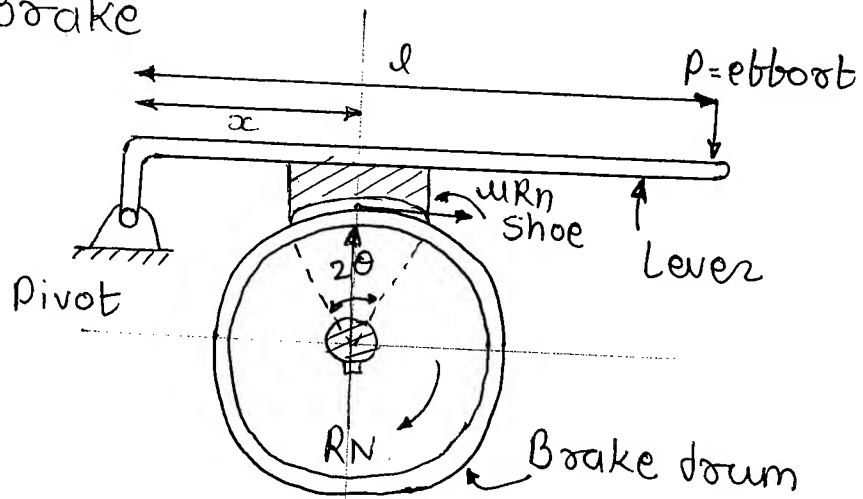
self energising

Movement of $P \times l$ and μR_N are in
same direction.

$$P = \frac{R_N (x - \mu a)}{l}.$$
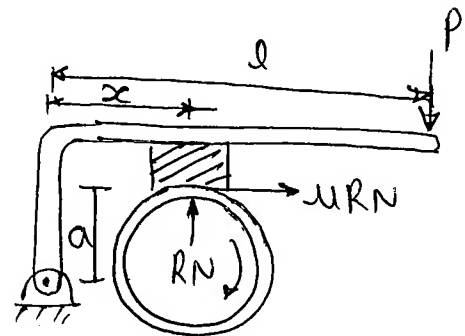
If $\mu a > x$
$\quad P - ve.$
$\quad R < 0$ self locking Brake.

To avoide self locking
$$x > \mu a$$
$$P \text{ is } + ve$$

l

x

P

a

R_N

μR_N

AM - Aplied movement
FM - Frichional movement

AM = FM = Same direchion
If same magnitude
self locking.

Case III

$$P \times l - R_N x - \mu R_N \times a = 0$$

AM (under $P \times l$), FM (under $\mu R_N \times a$)
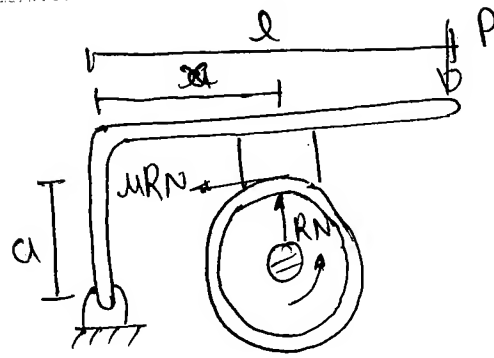
$$P = \frac{R_N(x + \mu a)}{l}$$

→ Not self energising

If $\mu a \geqslant x$ p is +ve

$P \leqslant 0$ — self locking brake.

Q:- If $\mu = 0.4$ and $x = 500\,mm$ what should be the value of a to get self locking.

$$x = \mu a$$

$$a = \frac{x}{\mu} = \frac{500}{0.4}$$

$$a = 1250\,mm.$$

$$P = \frac{R_N(x - \mu a)}{l}$$

Short Shoe.

$2\theta < 45°$ → Pressure is uniform

Long Shoe

$2\theta \geqslant 45°$ → Pressure in non uniform

$\mu' = \dfrac{4\mu \sin\theta}{2\theta + \sin2\theta}$ μ' = Virtual coff of friction

If $\theta = 0°$.

$\mu' = \dfrac{4\mu \sin0}{2 0° + \sin2 0°}$ $\sin\theta$ is very small $\sin\theta = \theta$

$= \dfrac{4\mu\theta}{2\theta + 2\theta} = \mu$

$\boxed{\mu' = \mu}$

If $\theta = \pi/2$.

$\mu' = \dfrac{4\mu \sin\pi/2}{2\times\dfrac{\pi}{2} + \sin2\times\dfrac{\pi}{2}} = \dfrac{4\mu}{\pi}$

$\mu' = 1\cdot273\mu$

$\boxed{\mu' = 1\cdot3\mu}$

$1 < \dfrac{\mu'}{\mu} < 1\cdot3$.

Band Brake

Rope (or) steel band coiled with

Brake down.
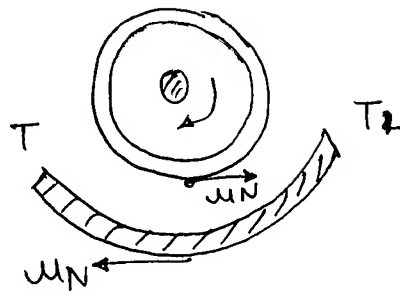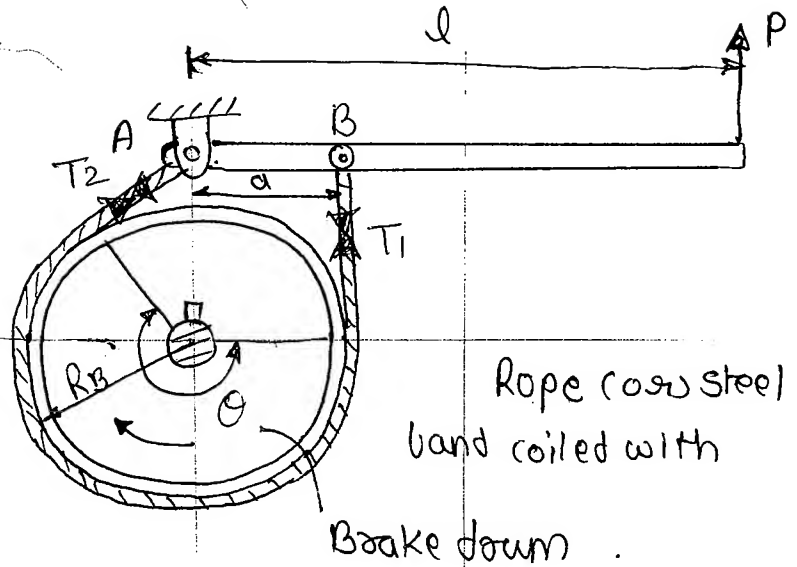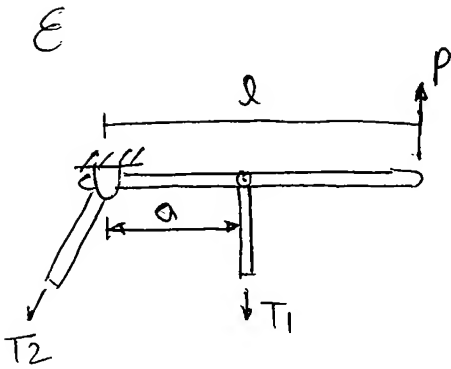
$$\Sigma M_{pivot} = 0.$$

case i) $P \times l - T_1 \times a = 0$ —① (↻) clockwise
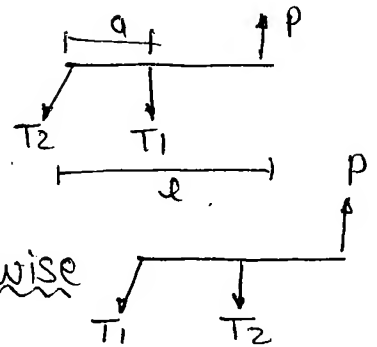
case ii) $P \times l - T_2 \times a = 0$ —② (↺) Counter clockwise

case ①
$$P = \frac{T_1 \times a}{l}$$

case ②i
$$P = \frac{T_2 \times a}{l}$$

Not self locking type.

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

(Breaking torque) $T_B = (T_1 - T_2) R_B.$

Simple band Brake.

One end of band is connected to pivot and the other end is connected to lever.

- Non self energising brake
- Not self locking brake.

Pressure is not uniform hence local wear take place in case of shoe brake, ware is uniform hence ~~shoe~~ brake band. is not used in automotive. In case of band brake
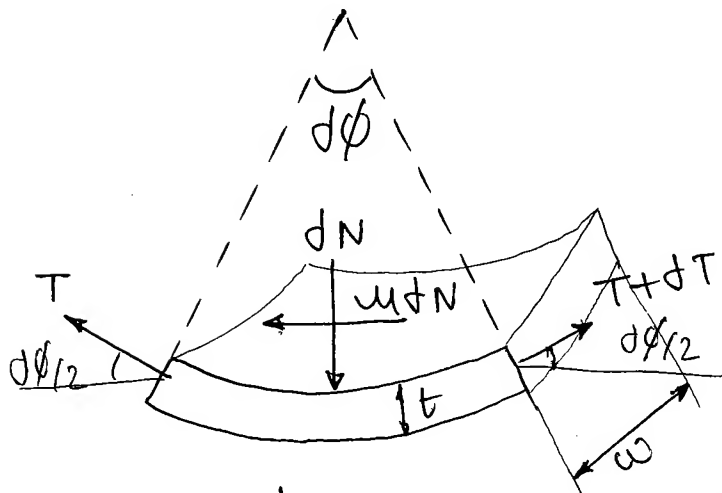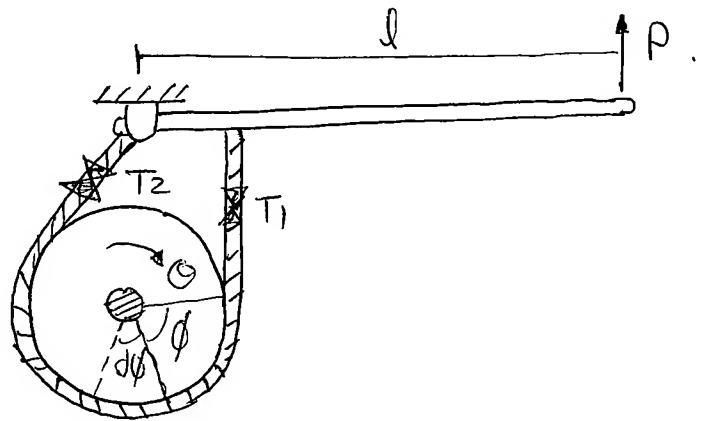
$$T \sin \frac{d\phi}{2} + (T + dT) \sin \frac{d\phi}{2} = dN.$$

$$d\phi \text{ is very small}$$

$$\sin \frac{d\phi}{2} = \frac{d\phi}{2}$$

$$T \frac{d\phi}{2} + \frac{d\phi}{2} = dN.$$

t = thickness

ω = width

$$Td\phi = dN. \quad -①$$

Normal force = Pressure × area

$$dN = P \times (R\,d\phi)\,\omega$$

$$dN = PR\,d\phi \times \omega$$

$$Td\phi = PR\,d\phi \times \omega$$

$$P = \frac{T}{\omega R}$$

P – pressure.

Max pressure
$$P_{max} = \frac{T_1}{WR}$$

Min Pressure
$$P_{min} = \frac{T_2}{WR}$$

Pg no 88

Qno 28 and 29.

$R = 500\,mm$

$F = 300\,N$.

$\mu = 0.45$.
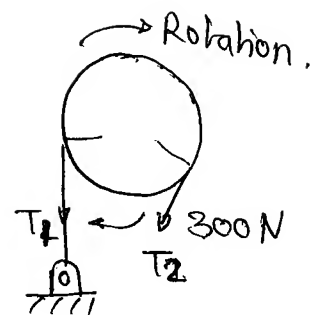
$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$T_B = 800\,N$$

$$T_B = (T_1 - T_2)\,R_B$$

$$800 = (\underset{T_1 - 300}{\cancel{300 - T_2}}) \times 500 \times 10^{-3}$$

$$T_1 = \cancel{301.6} \quad 1900.$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{1900}{300} = e^{0.45\theta}$$

$$\ln\frac{1900}{300} = 0.45\theta$$

$$\text{rad } 4.101 = \theta$$

$$\boxed{\theta = 235.01^\circ}$$

$$P_{max} = \frac{T_1}{WR} = \frac{1900}{30\times10^{-3}\times500\times10^{-3}}$$

$$= 126.66\times10^3 \text{ N/m}^2.$$
$$= 126.66 \text{ KN/m}^2.$$

Qno. 8月 1 and 2

Pg no 84

m = 1500 kg.
V = 108 km/hr.
S = 100 m
D = 80 cm

T_b = ?

$$T_b = (T_1 - T_2)\,R.$$

$$S = ut + \frac{1}{2}at^2.$$
$$v^2 - u^2 = 2as.$$

1500 kg
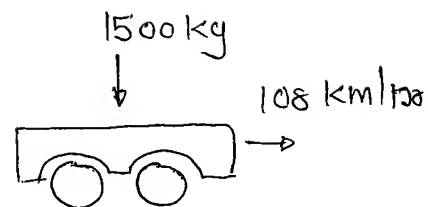
108 km/hr

$$K.E = \frac{1}{2}mv^2$$
$$= \frac{1}{2}\times1500\times\left(\frac{108\times1000}{60\times60}\right)^2$$
$$= 0.675\,\text{Kg}\,\frac{Km^2}{S^2}.$$

$$W.D = F_t\times S.$$
$$0.675 = F_t\times S.$$
$$10^6\times0.675 = F_t\times100$$
$$F_t = 6.75\times10^{+3} N.$$

$$T_b = F_t\times d/2 = 6.75\times10^{+3}\times\frac{80\times10^{-2}}{2}$$
$$= 2.7\times10^{+3} = 2.7 \text{ KN-m}$$

② $F_t = \mu R'.$ $\dfrac{F_t}{4} = \mu \dfrac{R}{4}$

$6.75 \quad \dfrac{2.7 \times 10^3}{4} = \mu \times 1500 \times 9.81$

$\boxed{\mu = 0.458}$.

Qno3 and 4

200   300

$\uparrow R \atop N'$

300N.

45° $F.$

150

100 rpm
D = 300 mm

$F \times 500 + -RN \times 200 \pm \mu RN \times 150 = 0$

$F \times 500 = RN \times 200$

$300 \times 500 = RN \times 200$

$RN = 750N$.

$T = \mu RN \dfrac{\ell}{2} = 0.24 \times 750 \times \dfrac{300 \times 10^{-3}}{2}$
$= 27\ N\text{-}m$
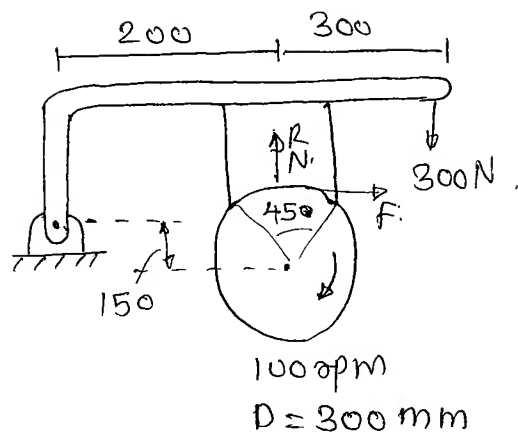
④ Heat generated

$Hg = T \times \theta$.
$= T \times \left(\dfrac{\omega_1 + \omega_2}{2}\right) \times t$.

$= 27\left(\dfrac{\dfrac{2\pi \times 100}{60}}{2} + 0\right) 5$

$= 706.85\ Joules$.

⑤.

$\mu' = \dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$

$= \dfrac{4 \times 0.3 \sin 45}{\left(\dfrac{\pi}{4}\right)2 \times 45 + \sin 90}$

do not put
45

put in rad. $\mu' = 0.330$

Q 6 to 9

$D_r = 650mm$.
$D_b = 1000mm$
load $= 2$ tonnes..
$V = 9kmph$.
$S = 2.75m$
$\mu = 0.2$ (Four shoe)
$P = 0.3 MPa$.
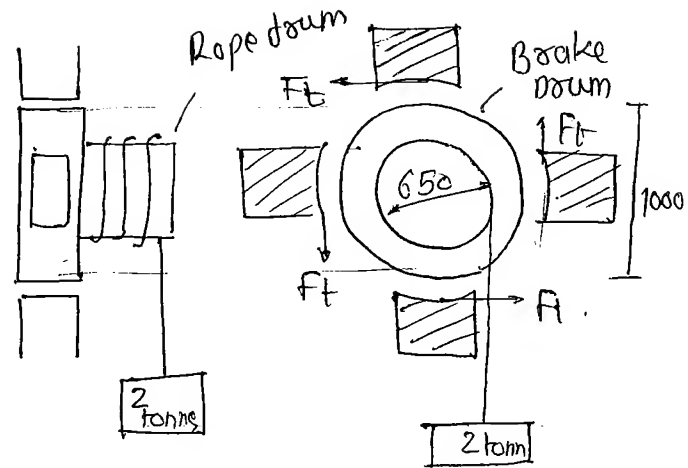
(6) $u = \dfrac{9 \times 1000}{60 \times 60} = 2.5 m/s$.

$v^2 - u^2 = 2aS$.

$0 - (2.5)^2 = 2 \times a \times 2.75$

$a = -1.13 \, m/s^2$.

$F_{total} = W - (ma)$.

$= 2000 \times 9.81 - (2000 \times -1.136)$.

$= 21892.72 \, N$.

$= 21.89 \, kN$.

$F = \dfrac{\mu N}{4}$.

(7) $torque = F_{total} \times \dfrac{D_r}{2} = F_t \times \dfrac{D_o}{2} \times 4$.

$\therefore F_{total} \dfrac{650}{2} \times 10^{-3} = F_t \times \dfrac{1000 \times 10^{-3}}{2} \times 4$

$\therefore 21.89 \times \dfrac{650 \times 10^{-3}}{2} = F_t \times \dfrac{1000 \times 10^{-3}}{2} \times 4$.

$\boxed{F_t = 3.55 \, kN}$

⑧.
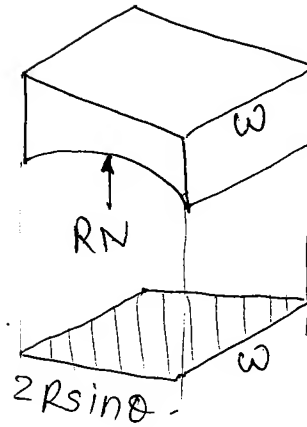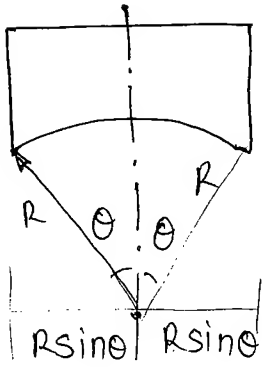
Pressure,

$$P = \frac{\text{Normal reaction}}{\text{Projected area}}$$

$$= \frac{RN}{\omega \times 2R\sin\theta} \qquad\qquad F_t = \mu RN$$

$$0.3 \times 10^6 = \frac{F_t/\mu}{\omega \times 2 \times 1000 \times 10^{-3} \times \sin\frac{45}{2}}$$

$$= \frac{17.175 \times 10^3}{\omega \times 2 \times \dfrac{1000 \times 10^{-3}}{2} \times \sin 45/2} \qquad \frac{N}{m^2} \quad \frac{N \times 10^6}{1000 \times 1000}$$

$$\omega = 0.1546 \, m$$

$$\boxed{\omega = 154.6 \, mm}$$

⑨.

Heat generated

$$Hg = T_6 \times \theta$$

$$= F_t \times \frac{D}{2} \times 4 \times \frac{\pi}{4}$$

$$= 3.55 \times \frac{1000 \times 10^{-3}}{2} \times 4 \times \frac{\pi}{2}$$

$$Hg/sec = \underbrace{F_t}_{\downarrow} \times V \qquad\qquad \text{or } T\omega$$

$$\mu RN$$

time req. for stopping.
$$V = u + at$$
$$0 = 2.5 - 1.136 t$$
$$t = 2.200 \text{ sec}$$

$$Hg = \mu RN \times v \times t$$

$$\theta = \left(\frac{w_1 + w_2}{2}\right) t$$
$$v = \frac{Dr}{2} \times w \qquad \boxed{V = RW}$$
$$\frac{m}{sec} 2.5 = \frac{650}{2} \times w$$
$$w_1 = 7.69 \text{ rad/sec}$$
$$w_2 = 0$$

$$T = F_{total} \times \frac{0.650}{2}$$
$$= 7.11 \text{ KN-m}$$

$$Hg = T \times \theta$$
$$= 7.11 \times \left(\frac{w_1 + w_2}{2}\right) t$$
$$= 7.11 \times \left(\frac{7.69 + 0}{2}\right) \times 2.2$$
$$= 60.14 \text{ KJ}$$

$$\cancel{\times} \text{ Torque (T)} = F_{total} \times R \times w \times t \quad \cancel{\times}$$
$$= 21.892 \times \frac{1000 \times 10^{-3}}{2} \times 54.6 \times 10^{-3} \times 2.2$$

Q 10 and 11

$$T = 450 \text{ N-m} \quad (\circlearrowright)$$

$\phi 150$

$220 N$

$50 \quad 100 \quad 100$

$T_1 \qquad T_2 \qquad 220 \text{ KN}$

$50 \quad 100 \qquad 100$

$$\sum M_{pivot} = 0$$
$$220 \times 200 \; \Box + T_1 \times 50 = T_2 \times 100$$
$$T_1 50 - T_2 100 = -220 \times 200 - \circled{1}$$

$$T = (T_1 - T_2) R$$
$$450 = (T_1 - T_2) \times \frac{150}{2}$$
$$\frac{450 \times 2}{150} = T_1 - T_2$$
$$T_1 - T_2 = \frac{450 \times 2 \times 10^3}{150} = 6 \times 10^3 \text{ N-m} \quad \circled{2}$$
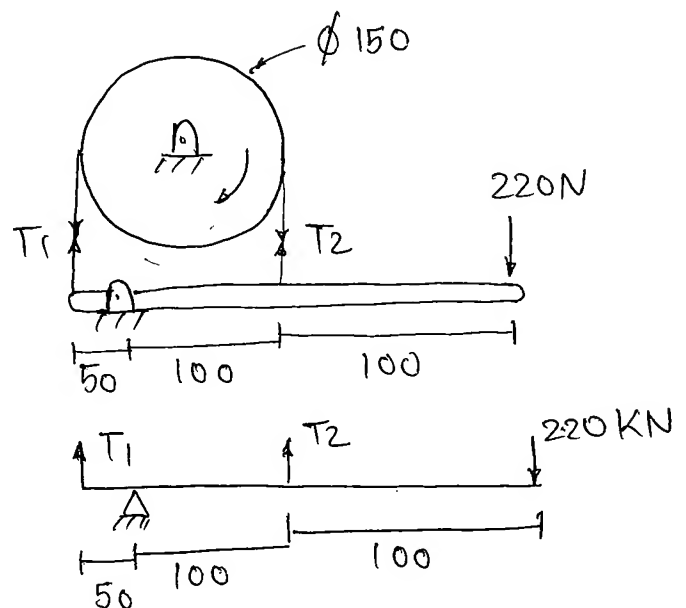
$$T_1 = 12800$$

$$T_2 = 6800$$

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$\frac{12800}{6800} = e^{\mu \times \frac{\pi}{180} \times 180}$$

$$\boxed{\mu = 0.29}$$

(11) $\mu = 0.4$

$$220 \times 200 + T_2 \times 50 = T_1 \, 100$$

$$T_1 \, 100 - T_2 \, 50 = 220 \times 200 \quad -①$$
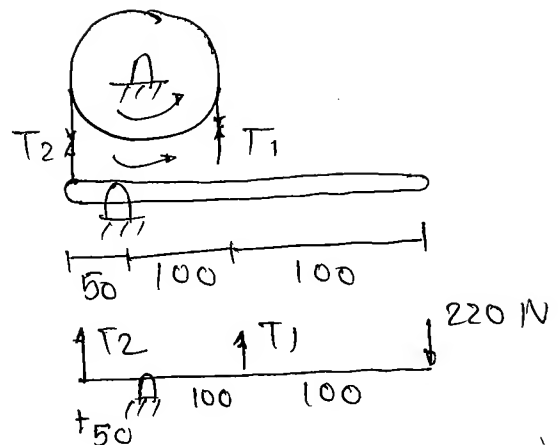
$$\frac{T_1}{T_2} = e^{\mu \theta}$$

$$\frac{T_1}{T_2} = e^{0.4 \times \pi}$$

$$T_1 = 3.51 \, T_2$$

$$T_2 = 146.179$$

$$T_1 = 513.089$$

$$T = (T_1 - T_2) \frac{150}{2}$$

$$= (513.089 - 146.179) \frac{150}{2}$$

$$= 27518.30 \; N\text{-}mm$$

$$T = 27.51 \; N\text{-}m$$

Q :- A force of 400N of is applied to the brake drum of 0.5m diameter in a band brake system as shown in fig when the wrapped angle is $180°$. If the coff. of friction is 0.25 the braking torque applied in N-m is .

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{0.25 \times 180 \times \frac{\pi}{180}}$$

$$T_1 = 2.19\, T_2 .$$

$$T_2 = 182.37 .$$

$$T_6 = (T_1 - T_2)\, D/2$$

$$= (400 - 182.37) \times \frac{0.5}{2}$$

$$T_b = \cancel{870.49}\ N\text{-}m$$
$$54.40 .$$
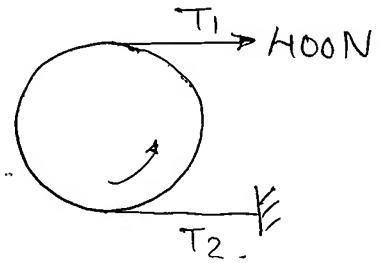
Direction of arrow in free space show T_1 direction

Differential Band Brakes.

$a > b$.

$a\theta > b\theta$.

$$\boxed{a > b}$$

P effort

Ball crank lever
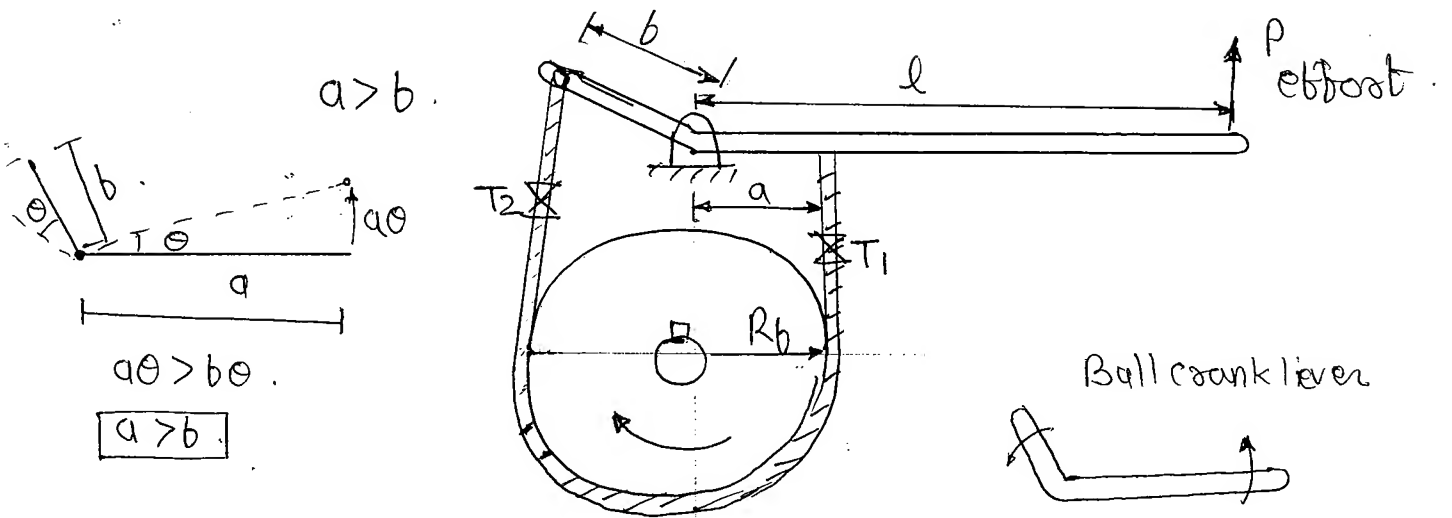
$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Torque , $T = (T_1 - T_2) R_b$.

ΣM pivot $= 0$.

$$P \times l - T_1 \times a + T_2 \times b = 0 .$$

$$T_1 a - T_2 b = Pl .$$

For self locking $P \, \text{\textcent} \, 0$.

$$T_1 a - T_2 b = 0 .$$

$$\frac{T_1}{T_2} = \frac{b}{a} = e^{\mu\theta}$$

$b\theta > a\theta$

$b > a$
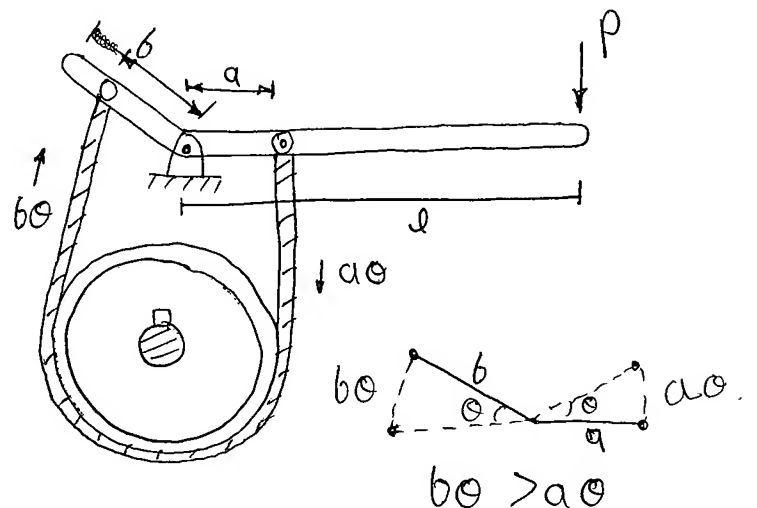
$b\theta$ is $>$ than $a\theta$ hence more

Internal Expanding Shoe Brake (Drum Brake).

$\sum M_{O_1} = 0$ (leading shoe)

$F \times l - N \times y + F_t \times x = 0$

Applied movement AM

Friction movemen. FM

The direction of AM and FM is same hence it is self energizing brake.

Take moment about O_2.

$\sum M_{O_2} = 0$.

$F \times l - N \times y - F_t \times x$

AM

FM

AM and FM are in opposite direction (Trailing shoe).

Brake drum

Cam

Friction material
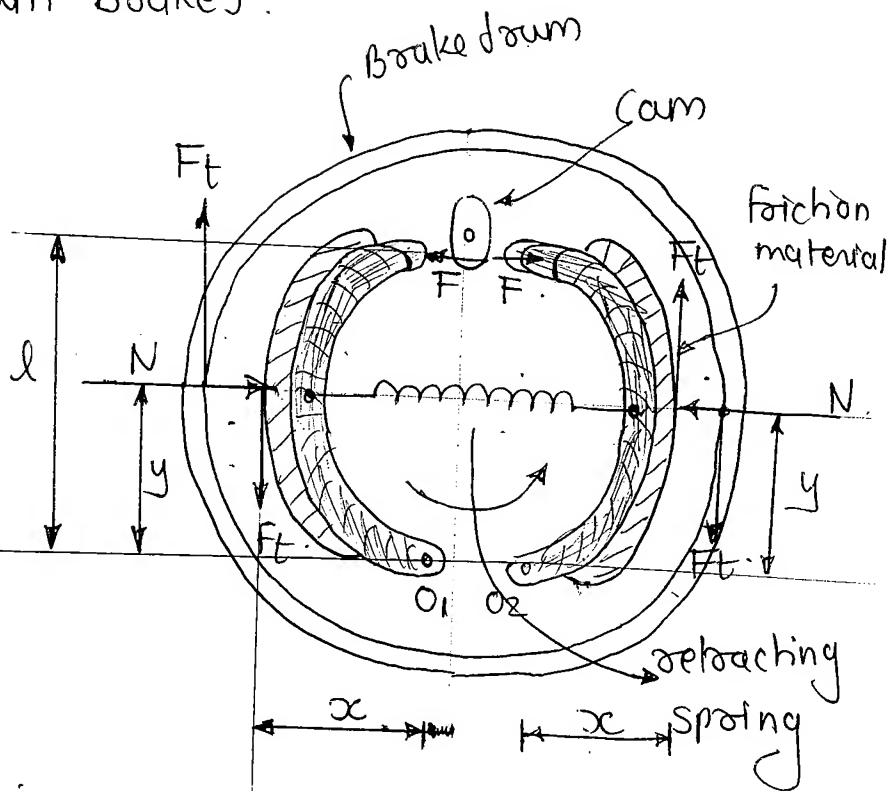
The direction of arrow between two shoe indicate leading or trailing shoe. arrow direction show the leading show direction between two shoe spare.
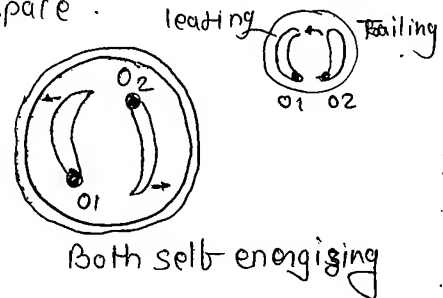
Both self-energizing

DESIGN OF CLUTCHES

A clutches is used to connect or disconnect the source of power from the remaining part of power transmition at the will of operator.
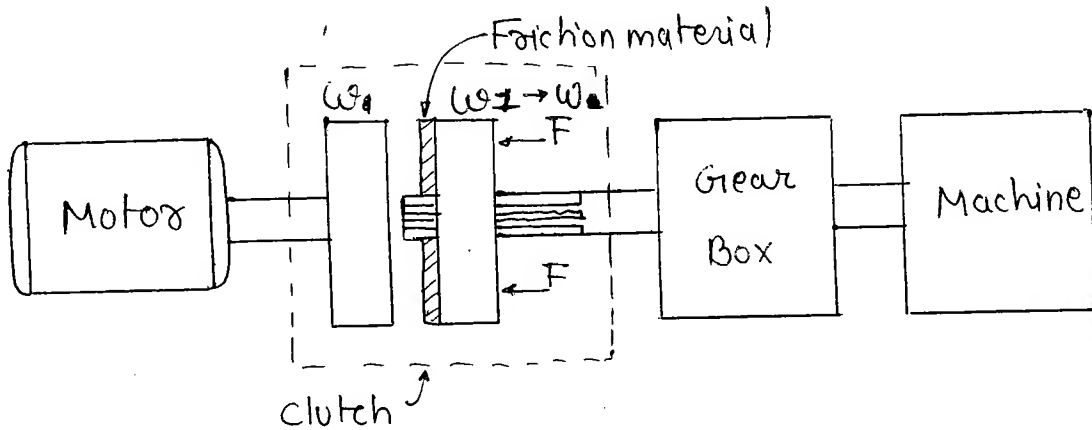
Amount of Heat generated (work done)

$$H_g = T \times \theta$$

$$\omega_2 = \omega_1 + \alpha t \quad \leftarrow v = u + \alpha t$$

$$\omega = 0 + \alpha t$$

$$\alpha = \frac{\omega}{t}$$

$$F = m \cdot a$$

Torque

$$T = I \alpha$$

$$= I \times \frac{\omega}{t}$$

$$v^2 - u^2 = 2as$$

$$\omega^2 - 0^2 = 2 \times \frac{N}{t} \times \theta$$

$$I = mk^2 \quad - \quad k - \text{Radius of gyration}$$

$$= \frac{D}{2} \text{ For a rimmed Flywheel}$$

$$= \frac{D}{2\sqrt{2}} \text{ For a Solid disc}$$

Types of clutches.

① Plate clutch
② Conical clutch
③ Centrifugal clutch.

Clutches → Friction → Friction force is used and there is slip.

Positive → No Friction force is used and no slip.
→ ① Square Jaw clutch ② Spiral Jaw clutch.

Spline shaft.
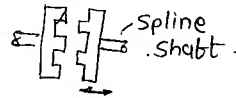
1] → Plate clutch

① Single plate
Ex:- Used in tractors, Buses, lorries and cars.

② Multiplate clutch.
Ex:- Machine tools, two wheelers.

2] → Conical clutch
Ex:- Feed drives of m/c tool.

3] → Centrifugal clutch
Ex:- Mopeds, washing m/c, coal crushers.

splineshaft
Square Jaw.

very slowspeed.

Spiral Jaw
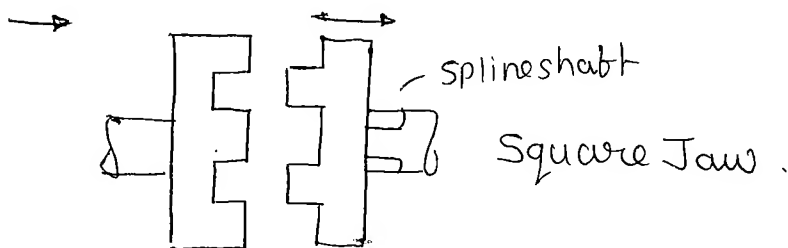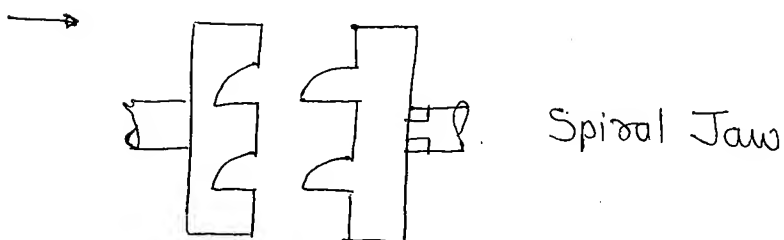
Bit high speed we can use this clutch.

2000rpm (High speed side)

motor —[clutch]— GB — m/c 5000rpm
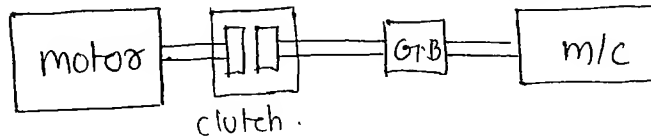
clutch.

$$P = \frac{2\pi NT}{60} \qquad P \propto NT.$$

$$N\uparrow \ T\downarrow.$$

As we req. more torque it we place it after th GB the size of the clutch will increse ($T \propto$ Radius of clutch) Hence we place clutch at high speed site ($N\uparrow \ T\downarrow$) which reduces the size of clutch.

Steel disc   spring   Effort.

Hub

clutch shaft

Crank shaft   collers

Pressure plate

Flywheel

P →   W $V_1 = r_1 \omega$

F_t

$V_2 = r_2 \omega$

P → r_1 r_2 F_t

F_t
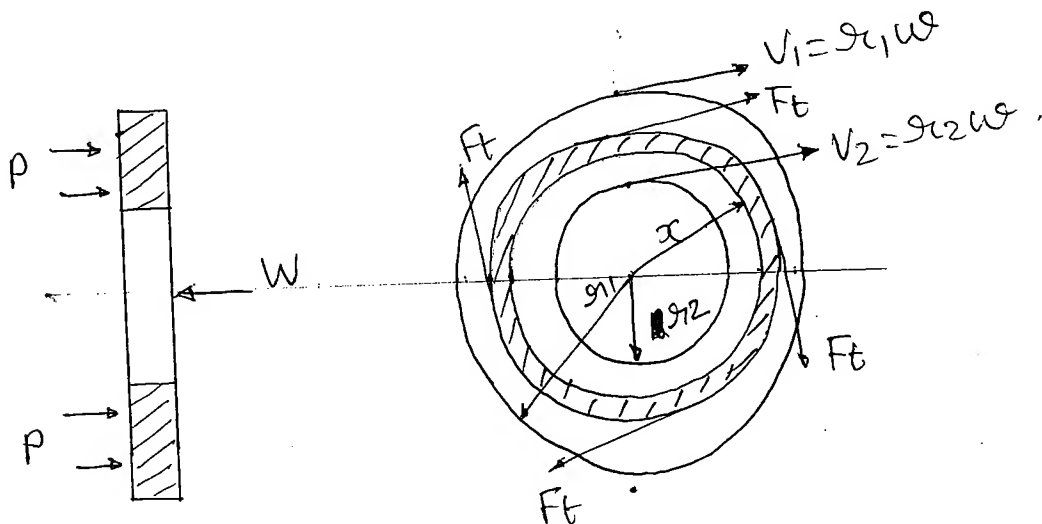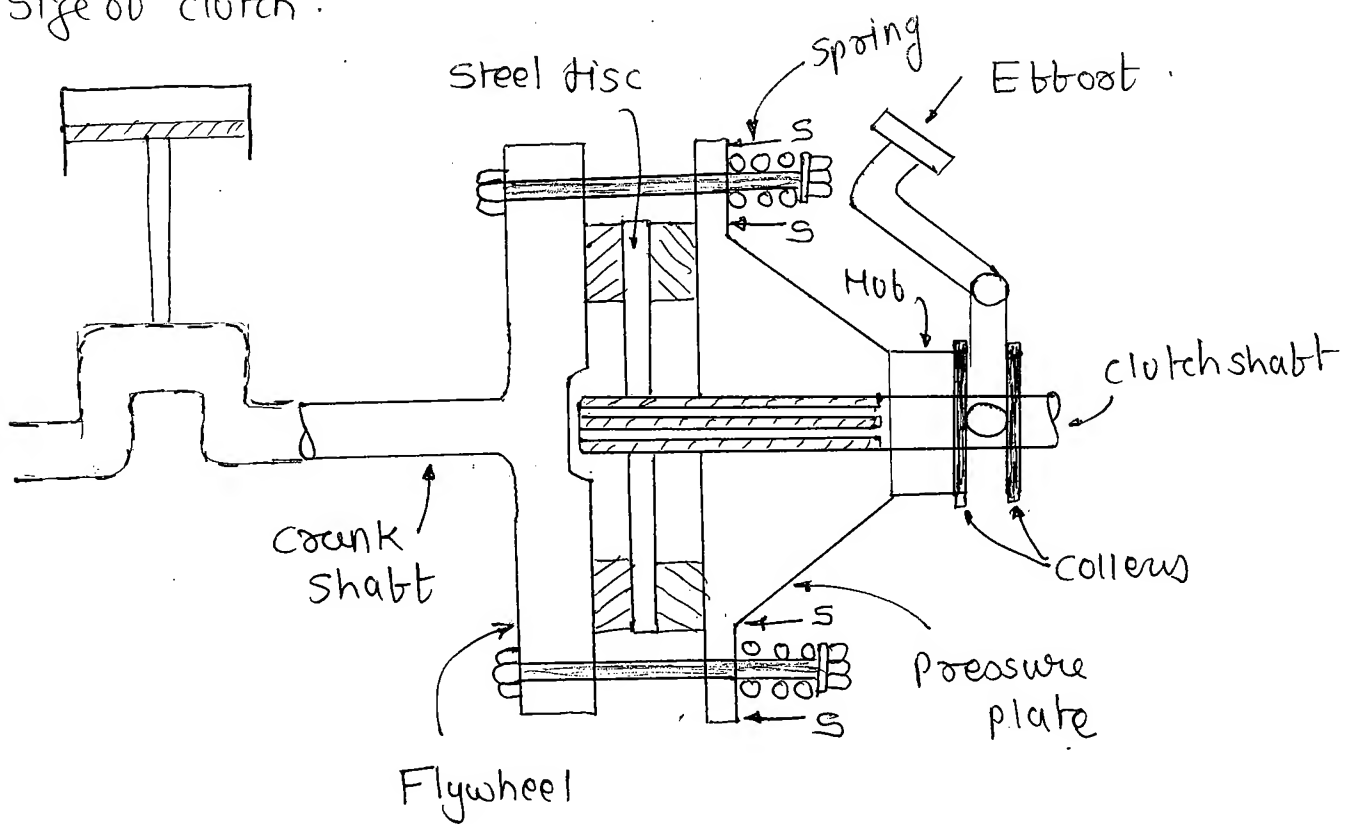
δw = Pressure × Area

$\quad = P \times 2\pi x \, dx$

Friction Force

$\quad \delta F = \mu \, \delta w$

$\quad\quad = \mu \, P \times 2\pi x \, dx.$

Torque

$\quad \delta T = \delta F \times x.$

$\quad\quad = \mu P \times 2\pi x^2 dx.$

Pressure is not uniform or may be uniform according to clutch condition or wear position.

1) New clutch — Uniform pressure

$$T = \int_{r_1} \delta T$$

$$= \int_{r_2}^{r_1} \mu \, P \, 2\pi x^2 \, dx.$$

$$= \mu P \, 2\pi \left[x^2 \, dx \right]_{r_2}^{r_1}$$

$$= 2\mu P \pi \left[\frac{r_1^3 - r_2^3}{3} \right]$$

$$= 2\pi \mu P \left[\frac{r_1^3 - r_2^3}{3} \right].$$

If $\delta w = P \, 2\pi x \, dx$

$$W = \int_{r_1}^{r_2} P \, 2\pi x \, dx$$

$$W = P\pi \left[r_1^2 - r_2^2 \right]$$

$$W = P\pi \left[r_1^2 - r_2^2 \right]$$

$$P = \frac{W}{\pi \left[r_1^2 - r_2^2 \right]}$$

$$= 2\pi \mu \times \frac{W}{\pi \left[r_1^2 - r_2^2 \right]} \left[\frac{r_1^3 - r_2^3}{3} \right]$$

$$= \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

② Used clutches — Uniform wear.

(Heat generated) $Hg = T \times \theta$.

$$Hg/sec = T \times \frac{\theta}{time}$$

$$'' \quad = T \times \omega$$

$$= \mu F \times r \times \omega$$

$$Hg/sec/mm^2 = \mu F V.$$

$$= \mu \times pressure \times velocity.$$

$$Hg/sec/mm^2 = \mu P V.$$

As   wear $\propto Hg/sec/mm^2$

$$\propto \mu P V$$

$$\propto P V$$

$\therefore$ If $PV = constant$ wear is uniform.

$$PV = constant$$

$$V = r\omega$$
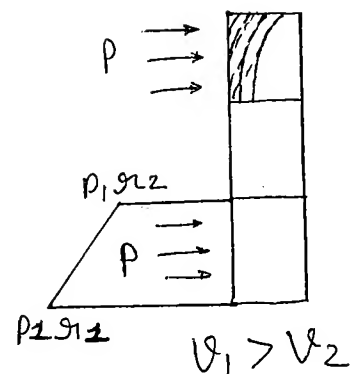
$$= x \omega$$

$$V \propto x.$$

$$Px = constant.$$

$$P_1 r_1 = P_2 r_2 = constant$$

$$P_{min} r_{max} = P_{max} r_{min}$$

$P \Rightarrow$

$P_1 r_2$

$P \Rightarrow$

$P_2 r_1$ $V_1 > V_2$

Initialy

$P_1 = P_2 \quad r_1 > r_2$

$P_1 r_1 > P_2 r_2$

After initial wear, wearing
take place uniform

$P_1 r_1 = P_2 r_2 = C$

$$\delta T = \mu P 2\pi x^2 dx$$

$$T = \int_{r_2}^{r_1} \delta T$$

$$= \mu 2\pi \int_{r_2}^{r_1} P x^2 dx$$

$$= \mu 2\pi C \int_{r_2}^{r_1} x \, dx$$

$$T = \mu 2\pi C \left(\frac{r_1^2 - r_2^2}{2} \right)$$

$$\delta \omega = P \times 2\pi x \, dx$$

$$\delta \omega = \frac{C}{x} 2\pi x \, dx$$

$$W = 2\pi C \int_{r_2}^{r_1} dx$$

$$C = \frac{W}{2\pi (r_1 - r_2)}$$

$$T = \mu \times 2\pi \times \frac{W}{2\pi (r_1 - r_2)} \times \left[\frac{r_1^2 - r_2^2}{2} \right]$$

$$T = \mu W \left(\frac{r_1 + r_2}{2} \right)$$

$$T = \mu W r_m \qquad\qquad r_m = \frac{r_1 + r_2}{2}$$

Tuniform press. $= \frac{2}{3} \mu W \left[\dfrac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$

Tuniform wear $= \mu W \left(\dfrac{r_1 + r_2}{2} \right)$

For shorter period of clutch life it subjected to uniform pr. after that its wear is uniform hence consider uniform wear when condition is not given

Uniform Pressure

$$T_{up} = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

Uniform wear

$$T_{uw} = \mu W \left(\frac{r_1 + r_2}{2} \right)$$

$$T_{up} > T_{uw}.$$

$$(r_F)_{up} > (r_F)_{uw}.$$

$$\frac{2}{3} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right] > \frac{r_1 + r_2}{2}$$
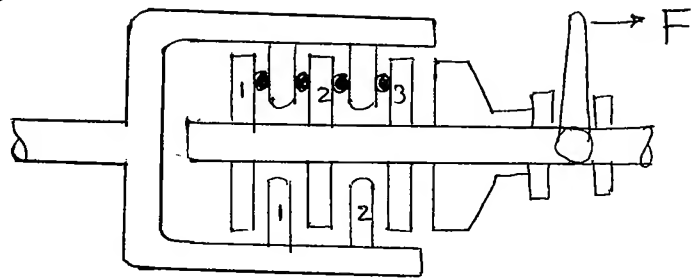
Multiplate clutch.

$$T_{wear} = \mu W \left(\frac{r_1 + r_2}{2} \right) n.$$

$n = $ no. of pairs of surface in contact

$$= n_1 + n_2 - 1$$
$$= 3 + 2 - 1$$
$$= 4.$$

Pno 76.
Q no 6 and 7

$r_F = 100 \, mm$

$\mu = 0.1$

$N = \dfrac{600}{\pi} \, rpm$

$W = 3 \, kN$

$T_{up} = ?$

$p = ?$

$$T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$= \mu W R_F.$$

$$= 0.1 \times 3 \times 100$$

$$= 30 \, N\text{-}mm$$

$$P = \frac{2\pi N T}{60} = \frac{2\pi \times 600/\pi \times 30}{60}$$

$$P = 600 \, W.$$

$n_1 = 5$.

$n = n-1 = 4$.

$T = 0.4$ KN-m

$= 0.4 \times 10^3$ N-m

$\mu = 0.5$

$r_1 = 150$ mm

$r_2 = \dfrac{200mm}{2} = 100$ mm

load/spring

$$T = \mu W \left(\dfrac{r_1 + r_2}{2}\right) n$$
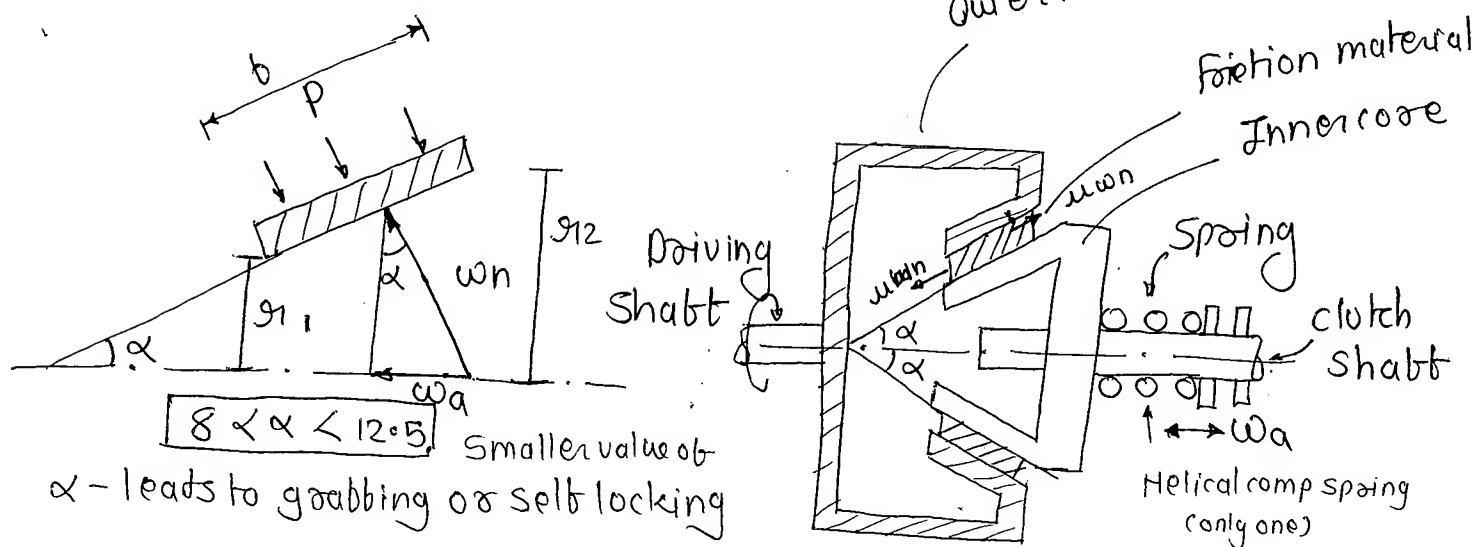
$$0.4 \times 10^3 = 0.5 \times W \left(\dfrac{150 \times 10^{-3} + 200 \times 10^{-3}}{2}\right) 4$$

$$W = \dfrac{0.4 \times 10^3}{0.35}$$

$$W = \dfrac{\cancel{1142.85}}{1600} \text{ N}.$$

$$\dfrac{load}{spring} = \dfrac{W}{no.\,of\,spring} = \dfrac{W}{4} = 400\,N.$$

Cone / Conical Clutch.

$8 < \alpha < 12.5$ Smaller value of

α - leads to grabbing or self locking

Helical comp spring (only one)

→ uniform pressure

$$T = \dfrac{2}{3} \mu W_n \left[\dfrac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \dfrac{2}{3} \mu \dfrac{W_n}{\sin\alpha}\left[\dfrac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

→ uniform wear

$$T = \mu W_n \left(\dfrac{r_1 + r_2}{2}\right) =$$

$$= \mu W_n \left(\dfrac{r_1 + r_2}{2}\right)$$

$$Sin\alpha = \frac{g_1 - g_2}{b}$$

$$Sin\alpha = \frac{W_a}{W_n}$$

$$W_a = W_n sin\alpha$$

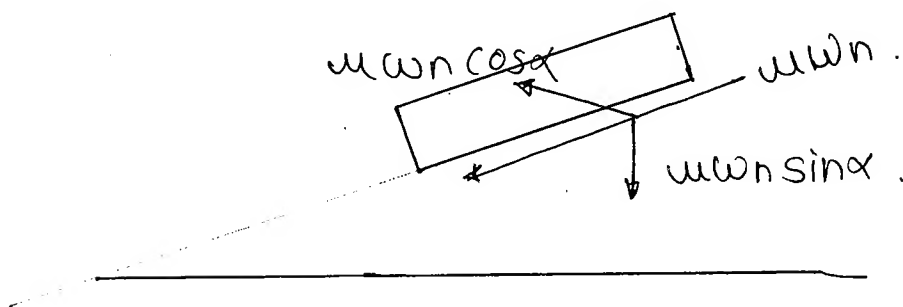W_a = Axial force applied during running.

$$T \propto \frac{1}{sin\alpha}$$

$$\alpha \downarrow \quad sin\alpha \downarrow \quad T \uparrow \cdots \cdots$$

Note -

① Driver and driven shaft must be colinear otherewise uneven wear take place.

②. Only one spring is used to balance the force even at reduced stiffness.

Axial force required to engauge the clutch.

$$W_{ae} = W_a + \mu W_n cos\alpha$$
$$= W_n sin\alpha + \mu W_n cos\alpha$$

$$W_{ae} = W_n (sin\alpha + \mu cos\alpha)$$

Q 13 to 14.

$P = 20 kw$

$N = 1000 rpm$

$r_b = \frac{r_1 + r_2}{2} = 150 mm$

$2\alpha = 24°$

$\alpha = 12°$

$\mu = 0.2$

$p = 70 \times 10^3 N/m^2$

$W_{ae} = W_n (\sin\alpha + \mu\cos\alpha)$

$= 6366.19 (\sin 12 + 0.2\cos 12)$

$W_{ae} = 2569.023$

$= 2.56 KN.$ Ans b]

$T = \mu W_n r_b$

$= 0.2 \times W_n \times 150 \times 10^{-3}$ — ①

$P = \frac{2\pi N T}{60}$

$T = \frac{P \times 60}{2\pi N}$ — ②

① = ②

$0.2 \times W_n \times 150 \times 10^{-3} = \frac{20 \times 10^3 \times 60}{2 \times \pi \times 1000}$

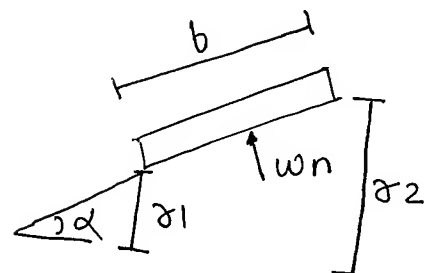$W_n = 6366.19 N.$

Pressure $= \frac{Normal\ force}{mean\ area\ of\ surface}$

$= \frac{W_n}{2\pi r_b}$

$70 \times 10^3 = \frac{6366.19}{2\pi \times 150 \times 10^{-3} \times b}$

$b = 0.0964 m.$

$b = 96.49 mm$

Ans b]

$r_b = r = \frac{r_1 + r_2}{2}.$

§ 38

$T_b = 140\,Nm$ $\quad\quad\quad T_b = (T_1 - T_2)R.$

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad\quad\quad 140 = (T_2 k - T_2) R$$

$$\frac{T_1}{T_2} = e^{0.2 \times \theta} \quad\quad 140 = T_2 (k-1) R.$$

$T_1 = k T_2.$ $\quad\quad -x-$

$$T = \mu\,\omega\left(\frac{r_1 + r_2}{2}\right) \quad = \mu \times 2\pi c\,(r_1 - r_2) \times \left(\frac{r_1 + r_2}{2}\right)$$

Case 1) 27.

$$C = P_{max}\,r_{min} = P_{min}\,r_{max}.$$
$$= P_{max}\,r_2 = P_{min} \times r_1$$

$$T = \mu \times 2\pi \times P_{max}\,r_2 \left(\frac{r_1^2 - r_2^2}{2}\right)$$

$$= \mu\,\pi\,P_{max}\,r_2\,(r_1^2 - r_2^2)$$

case 1) OD = 220mm, ID = 150mm, $P_{max} = 0.25\,MPa$ $\mu = 0.3$

$$T_b = 0.3 \times \pi \times 0.25 \times \frac{150}{2}\left(\left(\frac{220}{2}\right)^2 - \left(\frac{150}{2}\right)^2\right)$$

$$= 114.42 \times 10^3\, N\text{-}mm$$
$$= 114.42\, N\text{-}m < T$$

not ~~sota~~ recomended
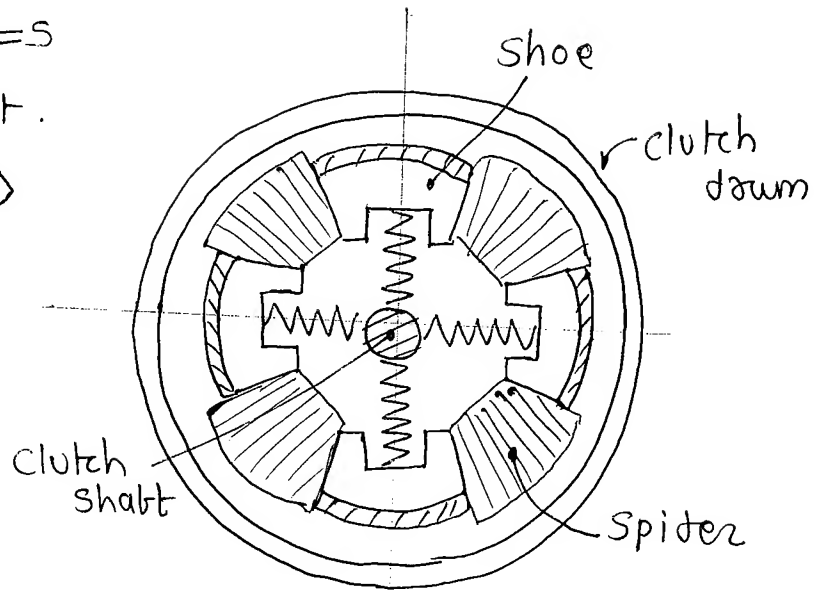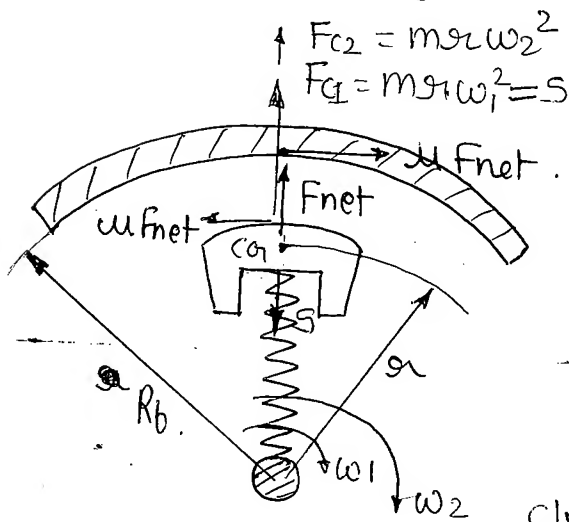
case 2] OD = 250mm, OD = 150, $P_{max} = 0.21\,MPa$ $\quad \mu = 0.3$

$T_b = 148\, N\text{-}M$ $\quad > T$ recommented (very good)

case 3] OD = 275mm, OD = 150mm, $P_{max} = 0.185$ $\mu = 0.3$

$T_b = 173\, N\text{-}m > T$ recommended

Centrifugal Clutch.

$F_{c2} = m \sigma \omega_2^2$

$F_{c1} = m \sigma \omega_1^2 = S$

μF_{net}

F_{net}

μF_{net}

R_b

σ

ω_1

ω_2

shoe

clutch drum

Clutch shaft

Spider

Provides time delay between starting of an engine and engagement of load

μ = coeff of friction

m = mass of each shoe

ω_1 = engageing speed

ω_2 = running speed

σ = distance between center of shaft to C.G of mass

R_b = Inner radius of clutch drum.

$F_{net} = F_{c2} - S$

$\qquad = F_{c2} - F_{c1}$

$\qquad = m \sigma (\omega_2^2 - \omega_1^2)$

Torque

T = Friction force × radius

$\quad = \mu F_{net} \times R_b$.

$\quad = \mu \times m \sigma (\omega_2^2 - \omega_1^2) \times R_b \times n$

n = no. of shoes.

Q.no 28

$T = 255 \, N\text{-}m$

$n = 4$

$\mu = 0.35$

$R_b = \dfrac{330}{2} = 165.$

$r_1 = 140 \, mm.$

$\omega_2 = 80 \, rad/sec$

$\omega_1 = 60 \, rad/sec$

$r = R_b - c$

$F_c = m r_1 \omega_1^2$
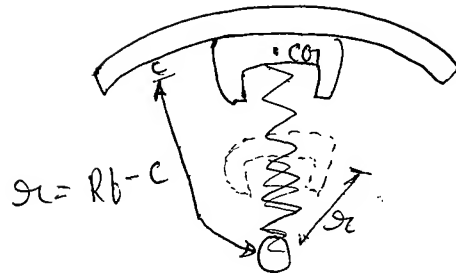
$F_{c2} = m r \omega_2^2$

$r \lll >>>>> c$

$T = \mu \, m_{r_1} (\omega_2^2 - \omega_1^2) \times R_b \times n.$

$255 = 0.35 \times m \times 140 \times 10^{-3} (80^2 - 60^2) \times 165 \times 10^{-3} \times 4$

$$\boxed{m = 2.48 \, kg}$$

GEARS

A gear is a toothed wheel used to transmit power at constant angular velocity ratio.

$$P = \frac{2\pi NT}{60}$$

$P \propto NT.$

$N \downarrow T \uparrow$

— Positive Drive (No slip)
— Short center distance.

Types of Gears.

1] Spur gear

① Tooth are cut parallel to axis

② Joins coplaner, parallel shaft.

Deep grooved Ball Bairing,
Cylindrical rollers
only radial load
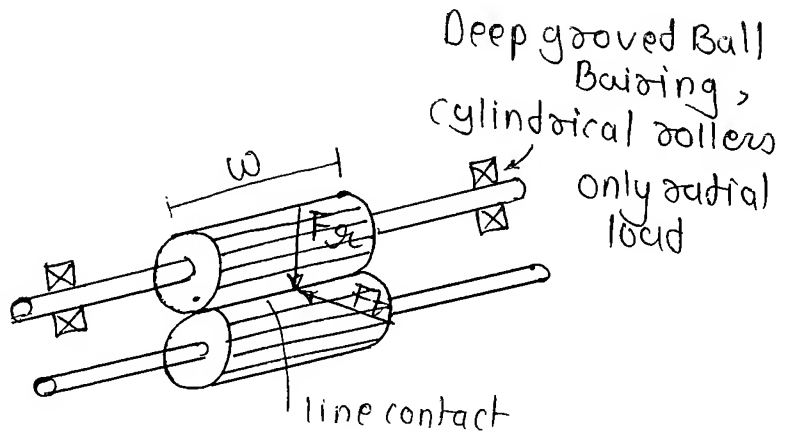
line contact

③ Line contact, sudden engagement and disengagement
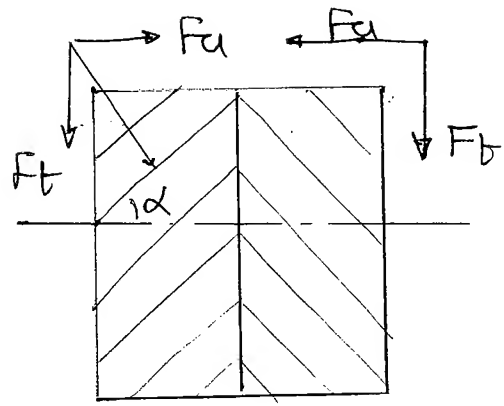
④ Only radial loads.

⑤ Velocity ratio 3:1 to 6:1

2] Helical gear

— Tooth are cut at an angle to the axis.

— Joins coplaner parallel shaft

— Initially point contact but gradually become line contact

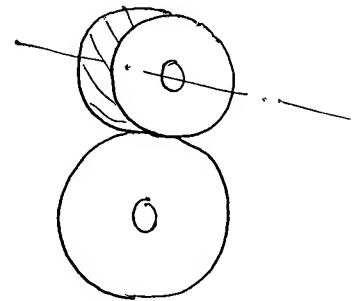— Velocity ratio 6:1 to 10:1

— loads both axial and radial

Tapered roller Bearing
Angle contact bearing
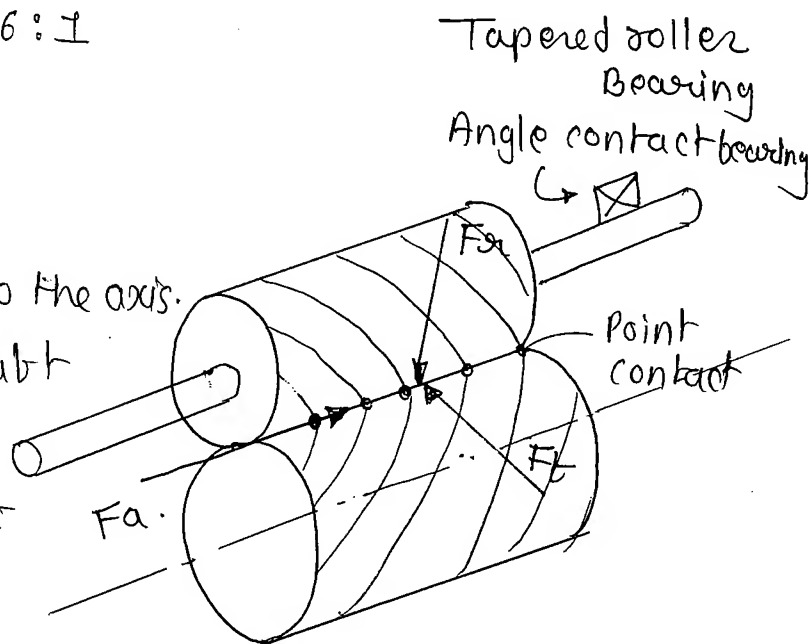
Point Contact
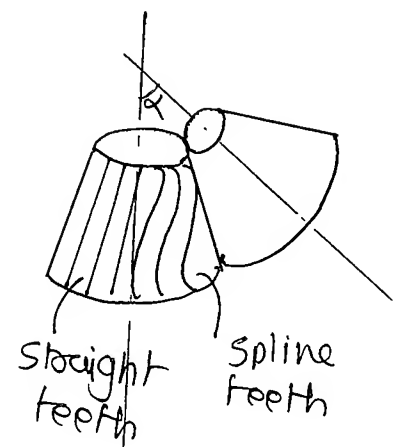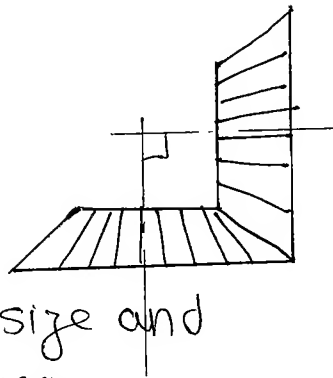
2] Herringbone gears.

axial force = 0.

3] Spiral gear / crossed helical gear.

- Joins non-coplenar and non intersecting shafts (skew shaft)
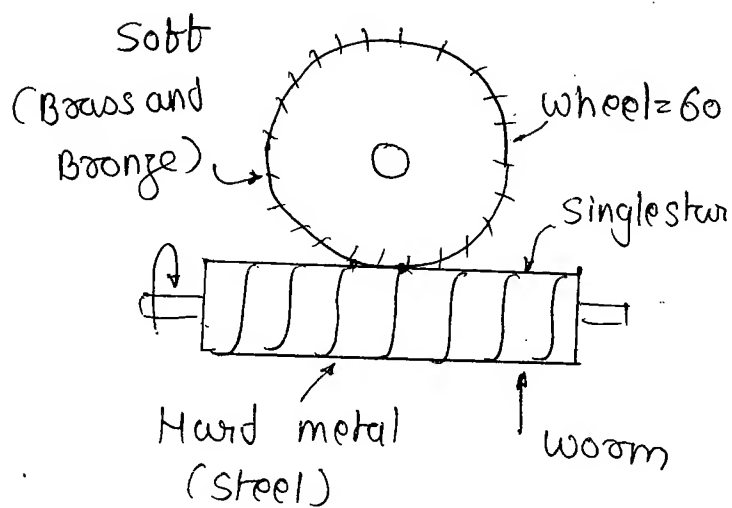
- Always point contact.

- Suitable for light loads.

4]

- Transmit power at angle
- Radial and axial loads
- Two bevel gear of equal size and at 90° are called mitre gear
- N line contact.

straight teeth   spline teeth
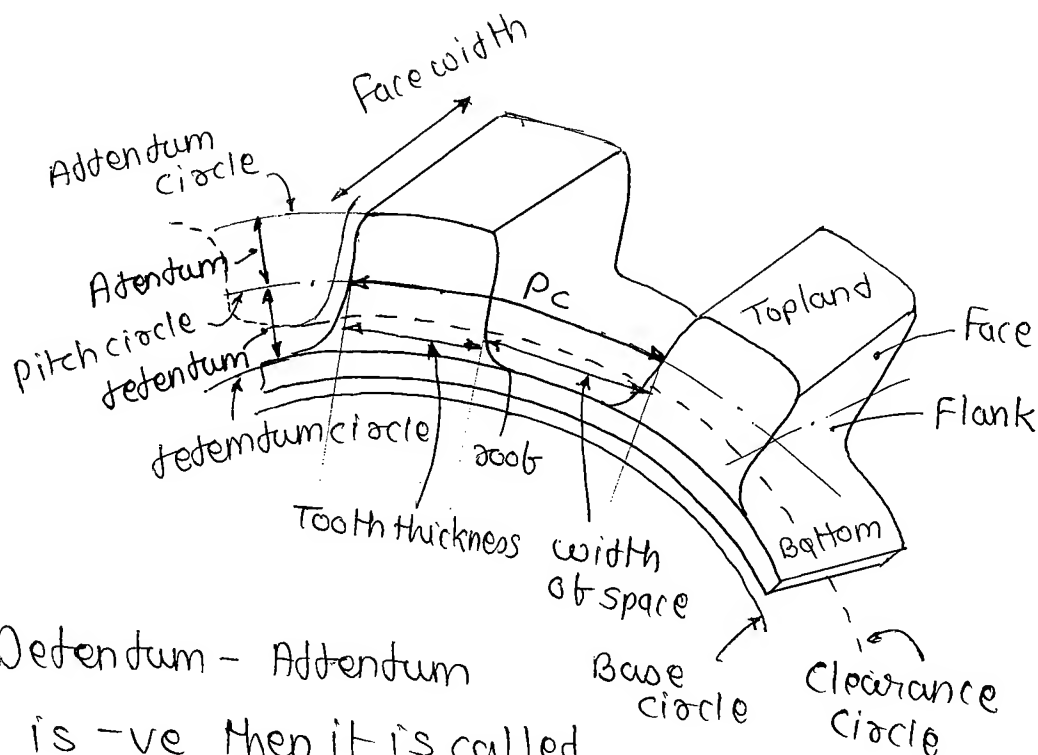
5] Woom gear

- Velocity ratio
  $$= \frac{1}{\text{No of teeth of wheel}}$$

- Skew shaft
- Velocity ratio upto 100 : 1.
- Initially point contact than line contact.

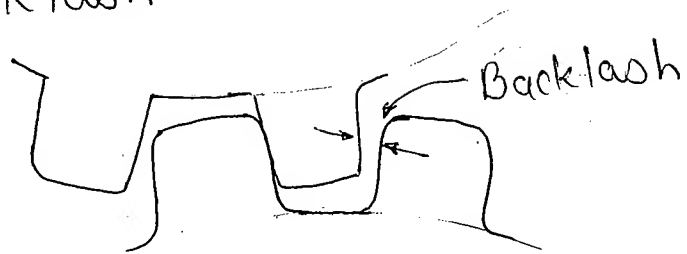Soft
(Brass and Bronze)

wheel= 60

Single star

Hard metal (Steel)

worm

SPUR GEAR.

Gear Nomenclature.

Face width

Addendum circle

Addendum

Pitch circle

Addendum

dedentum

dedentum circle

Tooth thickness

width of space

Pc

root

Topland

Face

Flank

Bottom

Base circle

Clearance circle

Clearance = Dedentum - Addendum
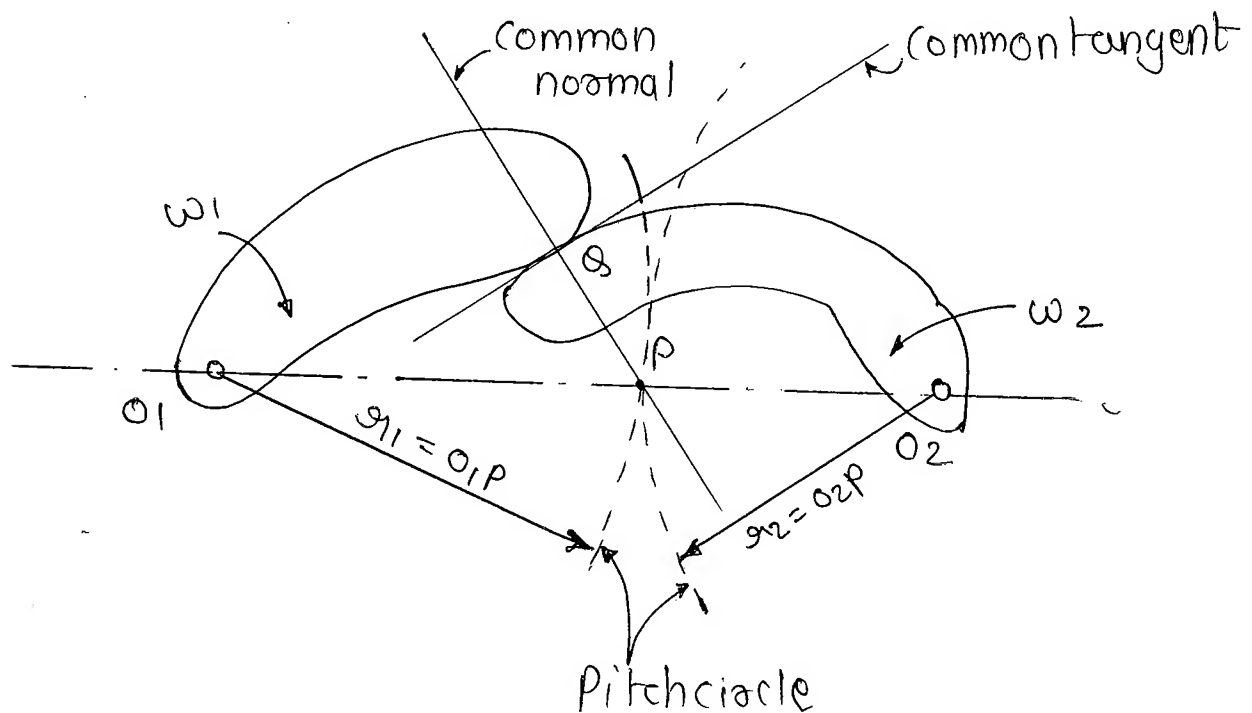If clearance is -ve then it is called
interbariance.

Backlash

Backlash

Back lash
  = Width of space - tooth thickness

Law of gearing.

The common normal at the point of contact always passes through a fixed point on the line joining centers and devites the line joining centers in the inverse ratio of thier angular velocities.

Pitch circle

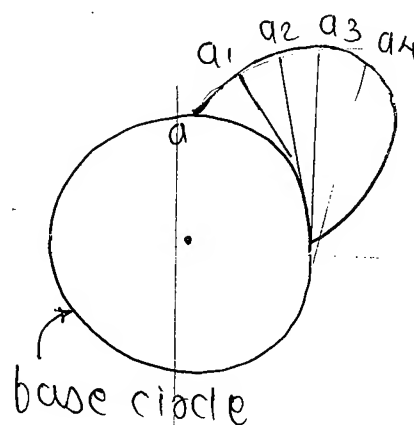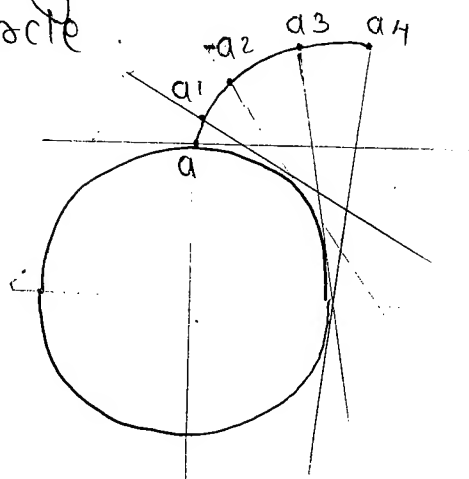$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{O_2P}{O_1P}$$

Pitch line velocity, $V = r_1\omega_1 = r_2\omega_2$

Conjugate Problem → (Involute , cycloid)

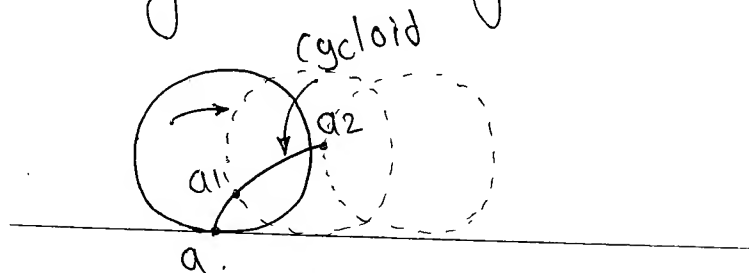- Satisfy low of gearing.
- Maintain constant angular velocity ratio.

Involute.

The locus of a point on a straight line which rolls without slipping on a circle is called involute circle is called base circle.
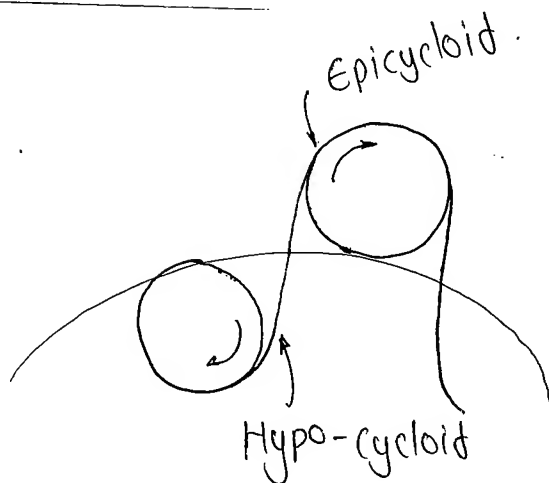

base circle

Cycloid

The locus of a point on a circle which rolls without slipping on a straight line is called cycloid.

cycloid

- More strength.
- Zero interference.
- Less ware.
- Difficult to cut

Epicycloid.

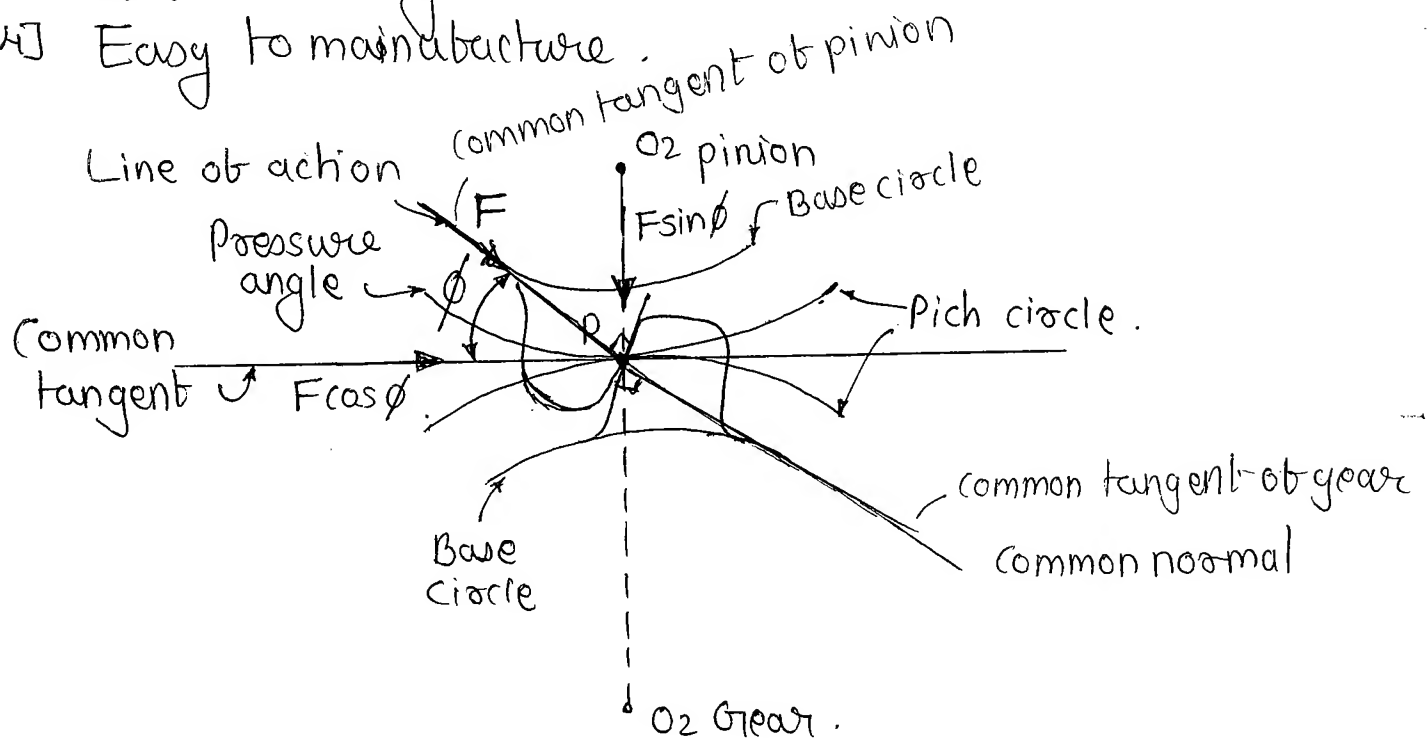Hypo-cycloid

Advantages of involute

1] Conjugate in action

2] Constant pressure angle (ϕ)
   (Pressure angle is zero at the pitch point and is maximum at tips For cycloid)

3] Small variation in the center distance will not effect velocity ratio.

4] Easy to manufacture.

Pressure angle (ϕ)

- Angle between common normal and common tangent
- Angle between common tangent to base circle and common tangent to pich circle
- Angle b/w line of action and common tangent is pressure angle.
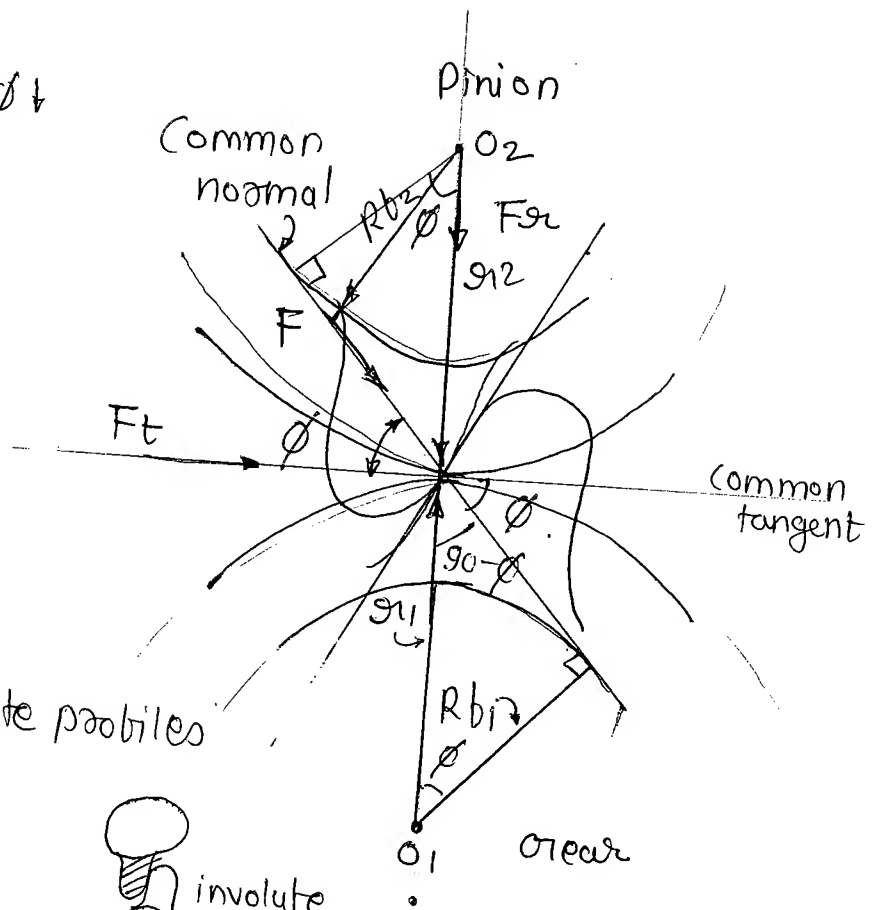
$$F_t = F\cos\phi$$
$$F_R = F\sin\phi$$
Torque, $T = F_t \times R$

$T\uparrow \ F_t \uparrow \ \cos\phi\uparrow \ \phi\downarrow$

$\phi = 14\frac{1}{2}$ obsolute
$= 20°$ – widely used
$= 25°$

$$F_R = F\sin\phi$$
$$F_R\downarrow \quad \sin\phi\downarrow \quad \phi\downarrow$$

Common normal

Common tangent

Pinion

O_2

Rb_2

ϕ

F_R

R_2

F

Ft

ϕ

$90-\phi$

R_1

Rb_1

ϕ

O_1

gear

Interference

① Negative clearance

② Meshing of non conjugate profiles

Involute

non involute

velocity ratio is not constant

involute
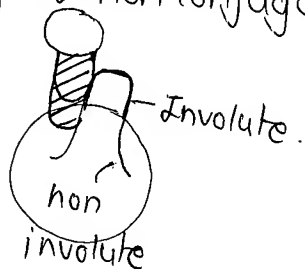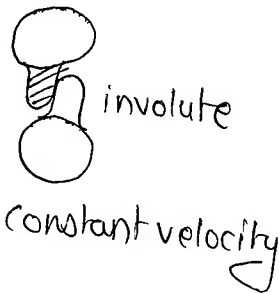
Constant velocity
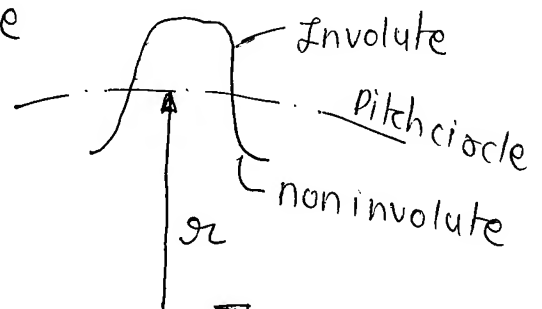
$$Ft = F\cos\phi$$
$$\phi\downarrow \quad Ft\uparrow$$

$$Rb = R\cos\phi$$
$$Rb \quad \phi\uparrow \quad \cos\phi\downarrow$$

we cant give or put $\phi = 0$

then $\boxed{Rb = R}$ then profile become

Involute

Pitch circle

non involute

R

$\phi = 20°$

$Rb = R\cos 20°$

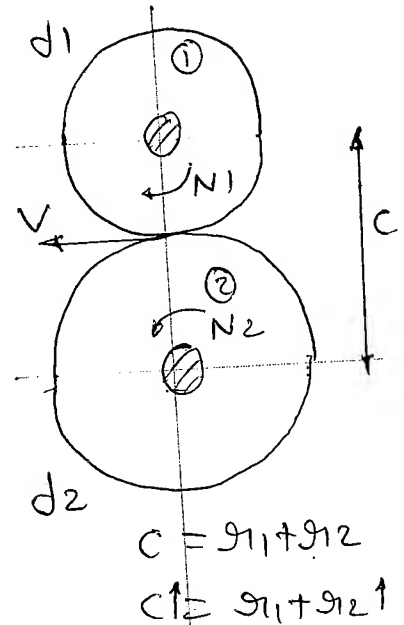$Rb = 0.939R$

This cause Interference hence velocity ratio is not constant

It $\phi = c$

③ Small variation. in the center distance does not change v.P
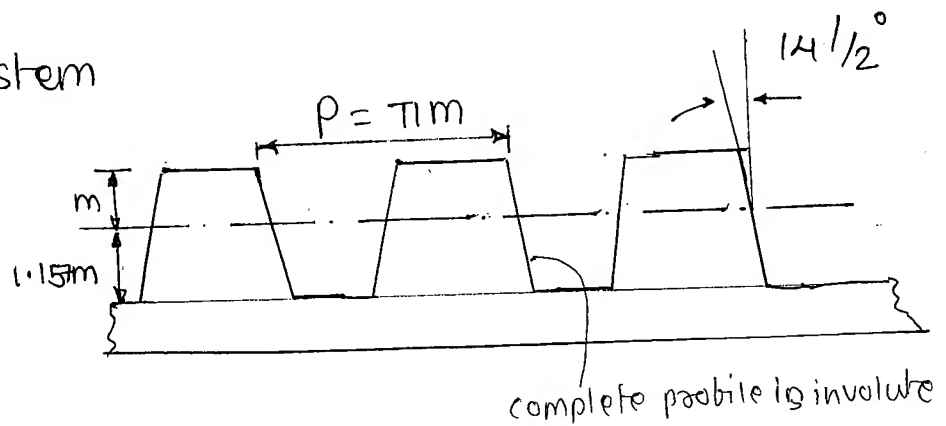
$$V = \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$$

$$V \cdot R = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{r_1}{r_2}$$

Ex:- $C = 100mm$ - design

but actual assembly time

$C = 98 mm$

$C = r_1 + r_2$

$C' = r_1 + r_2 \uparrow$

Gear System

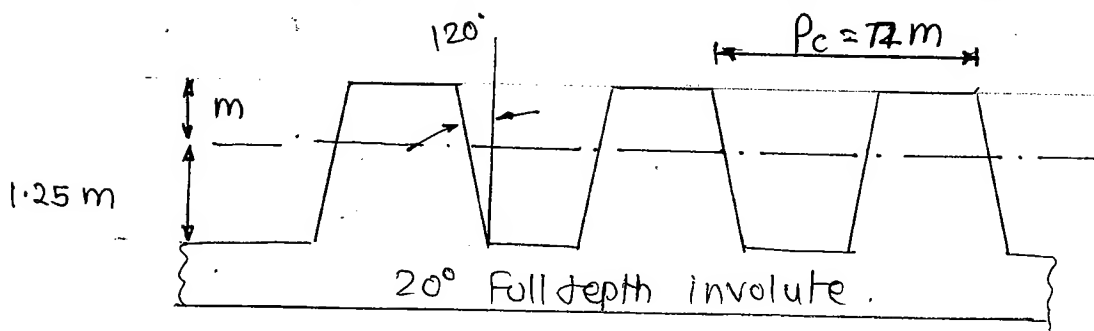
$P = \pi m$

$14\frac{1}{2}°$

complete profile is involute

① $14\frac{1}{2}$ Full depth involute

② → Pitch of gear

① Diametral pitch, $P_d = \dfrac{No. \ of \ teeth}{Pitch \ circle \ dia.} = \dfrac{T}{D}$ ~ is difficult

② Circular pitch, $P = \dfrac{\pi D}{T}$ ← difficult to express

D - Pitch circle dia
T - no. of teeth
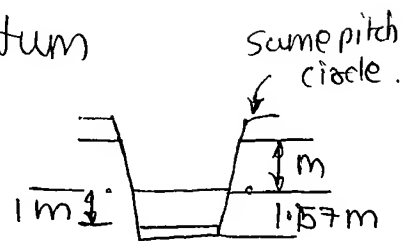
③ Module, $m = \dfrac{D}{T}$ mm

$\quad = \dfrac{1}{\wedge}$

Diagram: gear rack tooth profile showing $120°$, m, $1.25\,m$, $P_c = \pi\, m$, "20° Full depth involute."

① $14\frac{1}{2}$ Full depth involute

$$\text{Total depth} = \text{Addendum} + \text{Dedentum}$$
$$= m + 1.157\,m = 2.157\,m$$

$$\text{Working depth} = \text{addendum} + \text{addendum}$$
$$= 2m$$

Same pitch circle.

m / $1m$ / $1.57m$

$$\text{clearance}, \ e = 1.157\,m - 1m = 0.157\,m.$$

② $14\frac{1}{2}$ Composite teeth.

Cycloid

m

$1.157m$

To avoid interference
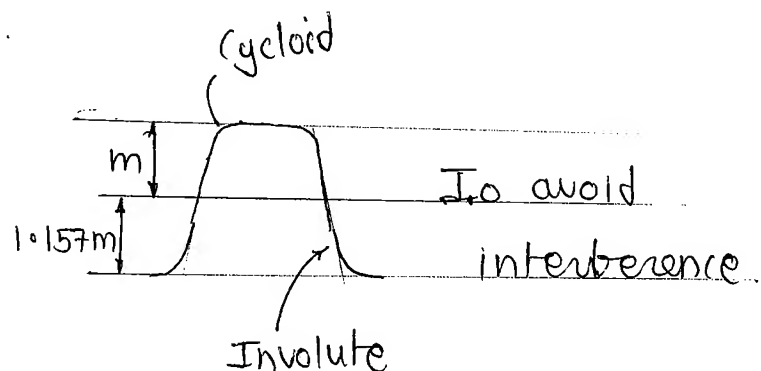
Involute
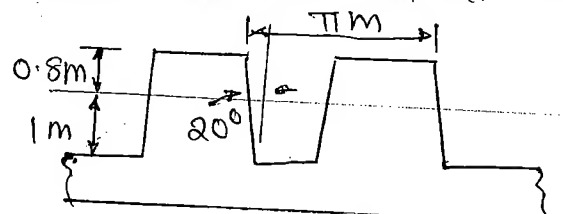
③ $20°$ Full depth involute

only clearance $= 0.25\,m$

④ $20°$ Stub involute

To reduce the bending stress because of high tooth hight in $20°$ Full depth we use $20°$ stub involute.
there is hight, limit to maintain the contact surface.

$$= 1 - 0.8$$
$$\text{Clearance} = 0.2\,m$$

$$\text{working depth} = 1.8\,m$$

Diagram: stub tooth profile showing πm, $0.8m$, $1m$, $20°$.

To avoid interference the minimum no of teeth on pinion

$$T_{min} = \frac{2}{\sin^2 \phi}$$

For $14\frac{1}{2}^\circ \rightarrow T_{min} = 31.9$

≈ 32

$20^\circ \rightarrow T_{min} = 17.09$

≈ 18

$25^\circ , \rightarrow T_{min} = 12$.

Tooth thickness

$$t = \frac{P_c}{2} = \frac{\pi m}{2} = \frac{\pi}{2} \times \frac{D}{T}.$$

$$t = \frac{\pi D}{2T}$$

$t \uparrow \quad T \downarrow \quad \phi \uparrow \quad \ldots \ldots \quad T_{min} \approx \frac{2}{\sin \phi}$

6-9 - PF
9:30-7 - MD
English
HT.

SAJIT CHINTAMKAR

Accepted that

NOT FALL IN LOVE WITH
ANY GIRL. IN LIFE

I HATE GIRLS*

I LOVE BOYS ONLY Sajit

3/6/ 2012
8. 10 AM

Dhanyawadam — Thanks.

Anshul.
9028372813

① Ni peranti   wats your name ?

Adhi ② A Wab pen   - My pen

③ Dine rate. anda - How waths the cost

④ Bhagunnava
   Bhagunava   - How you are fine .

⑤ Mirku ekto untana ru   ekkada where are you leaving loving

⑥ Mitho Frendship cheyacha. — I want to be your friend. make

⑦ Meru a college lo chatchadu vu thunna ru.

6-9 — Calculus
9.30-1 - M-S
2-3.36 - Production
6 - 8.30 thermal

$$ Z\theta = \dfrac{P\sin\theta\,(\cos\theta + \sin\theta)}{Sl} $$

$$ \dfrac{\partial Z}{\partial \theta} = 0 $$

$$ \dfrac{\partial}{\partial \theta}\left(\sin\theta\,(\cos\theta + \sin\theta)\right) $$

6-9 — PI
9-11 — TH
2-5.30 — TOM
6 - PI - PI

675°
45°

48°

48°/61°